same for all types of circuits and are different for different logic families (see Chapter 4). Unless otherwise specified, we shall be dealing with positive logic system.


Fig. 1.1 Digital Signal Representation (a) Positive Logic (b) Negative Logic

The above discussion brings out one of the main advantages of digital systems, viz. they are less susceptible to noise, fluctuations in the characteristics of components, etc.

The two discrete signal levels HIGH and LOW can also be represented by the binary digits 1 and 0 respectively. A binary digit ( 0 or 1 ) is referred to as a bit. Since a digital signal can have only one of the two possible levels 1 or 0 , the binary number system (see Chapter 2) can be used for the analysis and design of digital systems. The two levels (or states) can also be designated as ON and OFF or TRUE and FALSE. George Boole introduced the concept of binary number system in the studies of the mathematical theory of LOGIC in the work entitled An Investigation of the Laws of Thought in 1854 and developed its algebra known as Boolean algebra. These logic concepts have been adapted for the design of digital hardware since 1938 when Claude Shannon-organised and systematised Boole's work in Symbolic Analysis of Relay and Switching Circuits.

### 1.3 BASIC DIGITAL CIRCUITS

In a digital system there are only a few basic operations performed, irrespective of the complexities of the system. These operations may be required to be performed a number of times in a large digital system like digital computer or a digital control system, etc. The basic operations are AND, OR, NOT, and FLIP-FLOP. The AND, OR, and NOT operations are discussed here and the FLIP-FLOP, which is a basic memory element used to store binary information (one bit is stored in one FLIP-FLOP), will be introduced in Chapter 7.


Fig. 1.2 The Standard Symbol for an AND Gate

### 1.3.1 The AND Operation

A circuit which performs an AND operation is shown in Fig. 1.2. It has $N$ inputs $(N \geq 2)$ and one output. Digital signals are applied at the input terminals marked $A, B, \ldots, N$, the other terminal being ground, which is not shown in the diagram. The output is obtained at the output terminal marked $Y$ (the other terminal being ground) and it is also a digital signal. The AND operation is defined as: the output of an AND gate is 1 if and only if all the inputs are 1. Mathematically, it is written as

$$
\begin{align*}
Y & =A \text { AND } B \text { AND } C \ldots \text { AND } N \\
& =A \cdot B \cdot C \cdot \ldots \cdot N \\
& =A B C \ldots N \tag{1.1}
\end{align*}
$$

where $A, B, C, \ldots N$ are the input variables and $Y$ is the output variable. The variables are binary, i.e. each variable can assume only one of the two possible values, 0 or 1 . The binary variables are also referred to as logical variables.

Equation (1.1) is known as the Boolean equation or the logical equation of the AND gate. The term gate is used because of the similarity between the operation of a digital circuit and a gate. For example, for an AND operation the gate opens $(Y=1)$ only when all the inputs are present, i.e. at logic 1 level.

Truth Table Since a logical variable can assume only two possible values ( 0 and 1), therefore, any logical operation can also be defined in the form of a table containing all possible input combinations ( $2^{N}$ combinations for $N$ inputs) and their corresponding outputs. This is known as a truth table and it contains one row for each one of the input combinations.

For an AND gate with two inputs $A, B$ and the output $Y$, the truth table is given in Table 1.1. Its logical equation is $Y=A B$ and is read as " $Y$ equals $A$ AND $B$ ".

Table 1.1
Truth Table of a 2-Input AND Gate

|  | Inputs | Output |
| :---: | :---: | :---: |
| $\boldsymbol{A}$ | $\boldsymbol{B}$ | $\boldsymbol{Y}$ |
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

Since, there are only two inputs, $A$ and $B$, therefore, the possible number of input combinations is four. It may be observed from the truth table that the input-output relationship for a digital circuit is completely specified by this table in contrast to the input-output relationship for an analog circuit. The pattern in which the inputs are entered in the truth table may also be observed carefully, which is in the ascending order of binary numbers formed by the input variables. (See Chapter 2).

Logical Multiplication The AND operation is also referred to as logical multiplication and therefore, it is symbolised algebraically by a multiplication $\operatorname{dot}(\cdot)$ as illustrated in Eq. (1.1).

## Example 1.1

You have rented a locker in a bank. Express the process of opening the locker in terms of a digital operation.

## Solution

The locker door $(Y)$ can be opened by using one key $(A)$ which is with you and the other key $(B)$ which is with the bank executive. When both the keys are used, the locker door opens, i.e., the locker door can be opened ( $Y=1$ ) only when both the keys are applied $(A=B=1)$. Thus, this process can be expressed as an AND operation

$$
Y=A \cdot B
$$

## Example 1.2

The voltage waveforms shown in Fig. 1.3 are applied at the inputs of a 2 -input AND gate. Determine the output waveform.


Fig. 1.3

## Solution

Using Table 1.1, we find

From $t=0$ to $t=1$
From $t=\mathrm{x}$ to $t=4$
$A=1, B=0$
Therefore, $Y=0$
From $t=1$ to $t=2$
$A=B=1$
Therefore, $Y=1$
From $t=2$ to $t=3$
$A=1, B=0$
Therefore, $Y=0$
From $t=3$ to $t=\mathrm{x}$
$A=0, B=0$
Therefore, $Y=0$

The output waveform with reference to the input waveforms are shown in Fig. 1.4.


Fig. 1.4


Fig. 1.5 The Standard Symbol for ----- an OR Gate

### 1.3.2 The OR Operation

Figure 1.5 shows an OR gate with $N$ inputs ( $N \geq 2$ ) and one output. The OR operation is defined as: the output of an OR gate is 1 if and only if one or more inputs are 1 . Its logical equation is given by

$$
\begin{equation*}
Y=A \text { OR } B \text { OR } C \ldots \text { OR } N=A+B+C+\ldots+N \tag{1.2}
\end{equation*}
$$

The truth table of a 2-input OR gate is given in Table 1.2. Its logic equation is $Y=A+B$ and is read as " $Y$ equals $A$ OR $B^{\prime \prime}$.

Table 1.2 Truth Table of a 2-Input OR Gate

|  | Inputs | Output |
| :---: | :---: | :---: |
| $\boldsymbol{A}$ | $\boldsymbol{B}$ | $\boldsymbol{Y}$ |
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

## Example 1.3

In a chemical process an ALARM is required to be activated if either temperature or pressure or both exceed certain limits. Is it possible to express this operation in terms of a digital operation? If yes, find the operation.

## Solution

Let the temperature and pressure be converted into electrical signals and $T=1$ if temperature exceeds the specified limit and $P=1$ if pressure exceeds the specified limit. If $T=1$ or $P=1$ or both $T$ and $P$ are 1 then the ALARM is required to be activated, i.e., the signal applied to the ALARM $Y=1$. This operation can be expressed as

$$
\begin{aligned}
Y & =T \text { OR } P \\
& =T+P
\end{aligned}
$$

Which is an OR operation.

## Example 1.4

If the waveforms of Fig. 1.3 are applied at the inputs of a 2-input OR gate, determine the output waveform.

## Solution

Using Table 1.2, we find

| From $t=0$ to $t=1$ | From $t=2$ to $t=3$ |
| :--- | :--- |
| $A=1, B=0$ | $A=1, B=0$ |
| Therefore, $Y=1$ | Therefore, $Y=1$ |
| From $t=1$ to $t=2$ | From $t=3$ to $t=\mathrm{x}$ |
| $A=1, B=1$ | $A=0, B=0$ |
| Therefore, $Y=1$ | Therefore, $Y=0$ |

From $t=\mathrm{x}$ to $t=4 \quad$ From $t=5$ to $t=7$
$A=0, B=1$
Therefore, $Y=1$
$A=1, B=0$

From $t=4$ to $t=5$
$A=1, B=1$
Therefore, $Y=1$

The output waveform with reference to the input waveforms are shown in Fig. 1.6


Fig. 1.6

### 1.3.3 The NOT Operation

Figure 1.7 shows a NOT gate, which is also known as an inverter. It has one input $(A)$ and one output $(Y)$. Its logic equation is written as

$$
\begin{align*}
Y & =\text { NOT } A \\
& =\bar{A} \tag{1.3}
\end{align*}
$$



Fig. 1.7 The Standard Symbols for a NOT Gate
and is read as " $Y$ equals NOT $A$ " or " $Y$ equals complement of $A$ ". The truth table of a NOT gate is given in Table 1.3.

The NOT operation is also referred to as an inversion or complementation. The presence of a small circle, known as the bubble, always denotes inversion in digital circuits.

