

# Chapter 1

## Generalities

### 1.1 Units and Constants

Although many different units are employed in energy work, we shall adopt, whenever possible, the “Système International,” SI. This means **joules** and **watts**. If we are talking about large energies, we’ll speak of MJ, GJ, TJ, and EJ—that is,  $10^6$ ,  $10^9$ ,  $10^{12}$ , and  $10^{18}$  joules, respectively.

We cannot entirely resist tradition. Most of the time we will express pressures in **pascals**, but we will occasionally use **atmospheres** because most of the existing data are based on the latter. Sometimes **electron-volts** are more convenient than joules. Also, expressing energy in **barrels of oil** or **kWh** may convey better the idea of cost. On the whole, however, we shall avoid “quads,” “BTUs,” “calories,” and other non-SI units. The reason for this choice is threefold: SI units are easier to use, they have been adopted by most countries, and are frequently better defined.

Consider, for instance, the “calorie,” a unit preferred by chemists. Does one mean the “international steam table calorie” (4.18674 J)? Or the “mean calorie” (4.19002 J)? Or the “thermochemical calorie” (4.18400 J)? Or the calorie measured at 15 C (4.18580 J)? Or at 20 C (4.18190 J)?

Americans like to use the BTU, but, again, there are numerous BTUs: “steam table,” “mean,” “thermochemical,” at 39 F, at 60 F. The ratio of the BTU to the calorie of the same species is about 251.956 with some variations in the sixth significant figure. Remember that 1 BTU is roughly equal to 1 kJ, while 1 quad equals roughly 1 EJ. The conversion factors between the different energy and power units are listed in Table 1.2. Some of the fundamental constants used in this book are listed below.

**Table 1.1**  
**Fundamental Constants**

Quantity	Symbol	Value	Units
Avogadro’s number	$N_0$	$6.0221367 \times 10^{26}$	per kmole
Boltzmann constant	$k$	$1.380658 \times 10^{-23}$	J K <sup>-1</sup>
Charge of the electron	$q$	$1.60217733 \times 10^{-19}$	C
Gas Constant	$R$	8314.510	J kmole <sup>-1</sup> K <sup>-1</sup>
Gravitational constant	$G$	$6.67259 \times 10^{-11}$	m <sup>3</sup> s <sup>-2</sup> kg <sup>-1</sup>
Planck’s constant	$h$	$6.6260755 \times 10^{-34}$	J s
Permeability of free space	$\mu_0$	$4\pi \times 10^{-7}$	H/m
Permittivity of free space	$\epsilon_0$	$8.854187817 \times 10^{-12}$	F/m
Speed of light	$c$	$2.99792458 \times 10^8$	m s <sup>-1</sup>
Stefan-Boltzmann constant	$\sigma$	$5.67051 \times 10^{-8}$	W K <sup>-4</sup> m <sup>-2</sup>

**Table 1.2**  
**Conversion Coefficients**

To convert from	to	multiply by
<b>Energy</b>		
BARREL OF OIL	GJ	$\approx 6$
BRITISH THERMAL UNIT (Int. Steam Table)	joule	1055.04
BRITISH THERMAL UNIT (mean)	joule	1055.87
BRITISH THERMAL UNIT (thermochemical)	joule	1054.35
BRITISH THERMAL UNIT (39 F)	joule	1059.67
BRITISH THERMAL UNIT (60 F)	joule	1054.68
CALORIE (International Steam Table)	joule	4.18674
CALORIE (mean)	joule	4.19002
CALORIE (thermochemical)	joule	4.1840
CALORIE (15 C)	joule	4.1858
CALORIE (20 C)	joule	4.1819
CUBIC FOOT (Methane, STP)	MJ	$\approx 1$
ELECTRON VOLT	joule	$1.60206 \times 10^{-19}$
ERG	joule	$1.0 \times 10^{-7}$
FOOT LBF	joule	1.3558
FOOT POUNDAL	joule	$4.2140 \times 10^{-2}$
kWh	joule	$3.6 \times 10^6$
QUAD	BTU	$1.0 \times 10^{15}$
TON of TNT	joule	$4.2 \times 10^9$
<b>Power</b>		
FOOT LBF/SECOND	watt	1.3558
FOOT LBF/MINUTE	watt	$2.2597 \times 10^{-2}$
FOOT LBF/HOUR	watt	$3.7662 \times 10^{-4}$
HORSEPOWER (550 Foot LBF/sec)	watt	745.70
HORSEPOWER (electric)	watt	746
HORSEPOWER (metric)	watt	735
<b>Other</b>		
ATMOSPHERE	pascal	$1.0133 \times 10^5$
DALTON	kg	$1.660531 \times 10^{-27}$
LBF stands for pounds (force).		

## 1.2 Energy and Utility

In northern California, in a region where forests are abundant, one cord of wood sold in 1990 for about \$110. Although one cord is a stack of 4 by 4 by 8 ft (128 cubic feet), the actual volume of wood is only 90 cubic feet—the rest is empty space between the logs. Thus, one cord contains

1.2

2.5 m<sup>3</sup> of wood or about 2200 kg. The heat of combustion of wood varies between 14 and 19 MJ/kg. If one assumes a mean of 16 MJ per kilogram of wood burned, one cord delivers 35 GJ. Therefore, the cost of energy from wood was \$3.2/GJ in northern California.

In 1990, the price of gasoline was still approximately \$1.20 per gallon, the equivalent of \$0.49 per kg. Since the heat of combustion of gasoline is 49 MJ/kg, gasoline energy costs \$10/GJ, or three times the cost from burning wood.

Notwithstanding electricity being inexpensive in California, the domestic consumer paid \$0.04 per kWh or \$11.1/GJ.

From the above, it is clear that when we buy energy, we are willing to pay a premium for energy that is, in a more convenient form—that is, for energy that has a higher **utility**.

Utility is, of course, relative. To stoke a fireplace in a living room, wood has higher utility than gasoline and, to drive a car, gasoline has higher utility than electricity, at least for the time being. For small vehicles, liquid fuels have higher utility than gaseous ones. For fixed installations, the opposite is true.

The relative cost of energy is not determined by utility alone. One barrel contains 159 liters or 127 kg of oil. With a heat of combustion of 47 MJ/kg, this corresponds to 6 GJ of energy. In mid-1990, the price was \$12/barrel or \$2/GJ, somewhat less than the price of wood at that time notwithstanding oil being, in general, more useful. However, oil prices are highly unstable depending on the political circumstances of the world.

Government regulations tend to depress prices below their free market value. During the Carter era, natural gas was sold in interstate commerce at the regulated price of \$1.75 per 1000 cubic feet. This amount of gas corresponds to 1 GJ of energy. Thus, natural gas was cheaper than oil or wood.

### 1.3 Conservation of Energy

Energy can be utilized but not consumed.<sup>†</sup> It is a law of nature that energy is conserved. Instead of consuming it, we degrade or randomize energy, just as we randomize mineral resources when we process concentrated ores into metal and then discard the final product as we do, for example, with used aluminum cans. All energy we use is degraded into heat and eventually radiated out into space.

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<sup>†</sup> It is convenient to distinguish *consumption* from *utilization*. The former implies destruction—when oil is consumed, it disappears being transformed mainly into carbon dioxide and water, yielding heat. On the other hand, energy is never consumed—it is utilized but entirely conserved (only the entropy is increased).

The consumable is not energy; the consumable is the fact that energy has not yet been randomized. The degree of randomization of energy is measured by the entropy of the energy. This is discussed in some detail in Chapter 2.

## 1.4 Planetary Energy Balance

The relative stability of Earth's temperature suggests a near balance between planetary input and output of energy. The input is almost entirely that of the solar radiation incident on Earth. This amounts to 173,000 TW ( $173,000 \times 10^{12}$  W).

Besides solar energy, there is a contribution from tides (3 TW) and from heat sources inside the planet, mostly radioactivity (32 TW).

Some 52,000 TW (30% of the incoming radiation) is reflected back to the interplanetary space: it is the **albedo** of Earth. All the remaining energy is degraded to heat and re-emitted as long-wave infrared radiation. Figure 1.1 shows the different processes that take place in the planetary energy balance mechanism.

The recurrence of ice ages shows that the equilibrium between incoming and outgoing energy is oscillatory in nature. Some fear that the observed secular increase in atmospheric CO<sub>2</sub> might lead to a general heating of the planet resulting in a partial melting of the Antarctic glaciers and consequent flooding of sea level cities. The growth in CO<sub>2</sub> concentration is the result of the combustion of vast amounts of *fossil*<sup>†</sup> fuels and the destruction of forests in which carbon had been locked.

Solar radiation 173,000 TW	{	Direct reflection	52,000 TW (30%)	} Short-wave radiation	
		Direct conversion to heat	78,000 TW (45%)		
		Evaporation of water	39,000 TW (22%)		
		Wind & waves	3,600 TW (2%)		
		Photosynthesis	40 TW		
	{	Tides	3 TW	} Long-wave radiation	
		Geothermal {	Volcanos & hot springs		
			Rock conduction		
			32 TW		

**Figure 1.1** Planetary energy balance.

<sup>†</sup> Fuels derived from recent biomass, such as ethanol from sugar cane, do not increase the amount of carbon dioxide in the atmosphere—such fuels only recycle this gas.

## 1.5 The Energy Utilization Rate

The energy utilization rate throughout the ages can only be estimated in a rough manner. In early times, man was totally nontechnological, not even using fire. He used energy only as food, probably at a rate somewhat below the modern average of 2000 kilocalories per day, equivalent to 100 W. Later, with the discovery of fire and an improved diet involving cooked foods, the energy utilization rate may have risen to some 300 W/capita.

In the primitive agricultural Mesopotamia, around 4000 B.C., energy derived from animals was used for several purposes, especially for transportation and for pumping water in irrigation projects. Solar energy was employed for drying cereals and building materials such as bricks. Per capita energy utilization may have been as high as 800 W.

The idea of harnessing wind, water and fire to produce useful work is ancient. Wind energy has been in use to drive sailboats since at least 3000 B.C. and windmills were described by Hero of Alexandria around 100 A.D. Extensive use of windmills started in Persia around 300 A.D. and, only much later, spread to China and Europe.

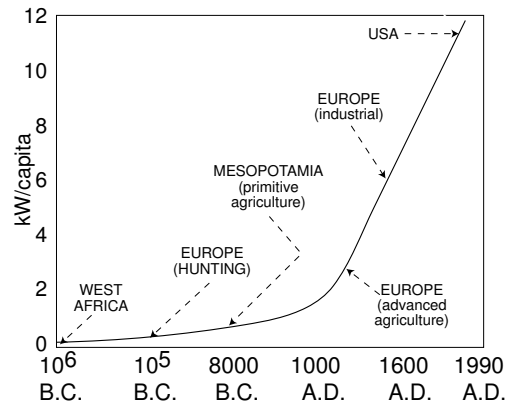
Hero described toy steam engines that apparently were built and operated. Vitruvius, the famous Roman architect and author whose book, first published at the time of Hero, is still on sale today, describes waterwheels used to pump water and grind cereals.

In spite of the availability of the technology, the ancients limited themselves to the use of human or animal muscle power. Lionel Casson (1981), a professor of ancient history at New York University, argues that this was due to cultural rather than economic constraints and that only at the beginning of the Middle Ages did the use of other energy sources become “fashionable.” Indeed, the second millennium saw an explosion of mechanical devices starting with windmills and waterwheels.

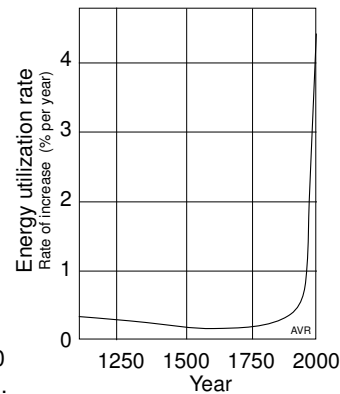
The energy utilization rate in Europe was likely 2000 calories per capita around 1200 A.D. when there was widespread adoption of advanced agriculture, the use of fireplaces to heat homes, the burning of ceramics and bricks, and the use of wind and water. Since the popular acceptance of such activities, energy utilization has increased rapidly.

Figure 1.2 illustrates (a wild estimate) the number of kilowatts utilized per capita as a function of the date. If we believe these data, we may conclude that the annual rate of increase of the per capita energy utilization rate behaved as indicated in Figure 1.3. Although the precision of these results is doubtful, it is almost certain that the general trend is correct—for most of our history the growth of the per capita energy utilization rate was steady and quite modest. However, with the start of the industrial revolution at the beginning of the 19th century, this growth accelerated dramatically and has now reached a worrisome level.

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**Figure 1.2** A very rough plot of the historical increase in the per capita energy utilization rate.



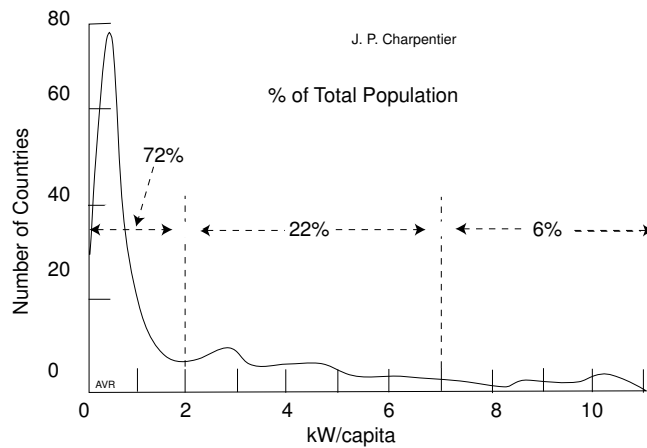
**Figure 1.3** The annual rate of increase of per capita energy utilization was small up to the 19th century.

One driving force behind the increasing worldwide per capita energy utilization was the low cost of oil before 1973 when the price of oil was substantially lower than what it is currently.<sup>†</sup> Perez Alfonso, the Venezuelan Minister of Oil in 1946, was among those who recognized that this would lead to future difficulties. He was instrumental in creating OPEC in 1954, not as a cartel to squeeze out higher profits but to “reduce the predatory oil consumption to guarantee humanity enough time to develop an economy based on renewable energy sources.” Alfonso also foresaw the ecological benefits stemming from a more rational use of oil.

OPEC drove the oil prices high enough to profoundly alter the world economy. The result was that the overall energy utilization rate slowed its increase. Owing to the time delay between the price increase and the subsequent response from the system, several years elapsed before a new equilibrium was established in the oil markets. The result was a major overshooting of the oil producing capacity of OPEC and the softening of prices that we witnessed up to the 1991 Iraqi crisis.

The recent effort of less developed countries (LDCs) to catch up with developed ones has been an important factor in the increase in energy demand. Figure 1.4 shows the uneven distribution of energy utilization rate throughout the world. 72% percent of the world population uses less than 2 kW/capita whereas 6% of the population uses more than 7 kW/capita.

<sup>†</sup> In 1973, before the OPEC crisis, petroleum was sold at between \$2 and \$3 per barrel. The price increased abruptly traumatizing the economy. In 2000 dollars, the pre-1973 petroleum cost about \$10/bbl (owing to a 3.8-fold currency devaluation), a price that prevailed again in 1999. However, in 2004, the cost had risen to over \$50/bbl.



**Figure 1.4** Most countries use little energy per capita while a few developed ones use a lot.

There is a reasonable correlation between the total energy utilization rate of a country and its corresponding annual gross national product. About 2.2 W are used per dollar of yearly GNP. Thus, to generate each dollar, 69 MJ are needed. These figures, which are based on 1980 dollars, vary with time, in part owing to the devaluation of the currency, but also due to changing economic circumstances. In fact, it has been demonstrated that during an energy crisis, the number of megajoules per dollar decreases, while the opposite trend occurs during financial crises.

Further industrialization of developed countries may not necessarily translate into an increase of the per capita energy utilization rate—the trend toward higher efficiency in energy use may have a compensating effect. However, in the USA, the present decline in energy utilization<sup>†</sup> is due mainly to a change in the nature of industrial production. Energy intensive primary industries (such as steel production) are phasing out owing to foreign competition, while sophisticated secondary industries (such as electronics and genetic engineering) are growing.

Technological innovation has resulted in more efficient use of energy. Examples of this include better insulation in houses and better mileage in cars. Alternate energy sources have, in a small measure, alleviated the demand on fossil fuels. Such is the case of using ethanol from sugar cane for the propulsion of automobiles. It is possible that the development of fusion reactors will, one day, bring back the times of abundant energy.

Introduction of a more efficient device does not immediately result in energy economy because it takes a considerable time for a new device to

<sup>†</sup> The use of energy by the American industry was less in 1982 than in 1973.

be widely accepted. The reaction time of the economy tends to be long. Consider the privately owned fleet of cars. A sudden rise in gasoline price has little effect on travel, but it increases the demand for fuel efficiency. However, car owners don't rush to buy new vehicles while their old ones are still usable. Thus, the overall fuel consumption will only drop many years later, after a significant fraction of the fleet has been updated.

Large investments in obsolete technologies substantially delay the introduction of more desirable and efficient systems. A feeling for the time constants involved can be obtained from the study of the "market penetration function," discussed in Section 1.7.

## 1.6 The Population Explosion

In the previous section we discussed the *per capita* energy utilization rate. Clearly the total rate of energy utilization is proportional to the planetary population which has been growing at an accelerated rate.<sup>†</sup>

The most serious problem that confronts mankind is the rapid growth in population. The planet has a little more than 6 billion inhabitants, and the growth rate these last few decades has been around 1.4% per year. Almost all projections predict a population of about 7 billion by the year 2010. This will be the case even if, right now, everyone were to agree on a limit of two children per family. Under present-day actuarial conditions, the population would eventually stabilize at around 11 billion by the year 2050. Thus, population growth alone could account for 1.4% a year increase in energy demand, in the next few decades.

If, in 2050, all the estimated 11 billion inhabitants of Earth were to use energy at the present day USA level (11 kW/capita), the world energy utilization rate would reach 122 TW—a 16-fold increase over the present 7.6 TW. Such a rate is probably one order of magnitude higher than can be supplied unless fusion energy becomes practical and inexpensive.

A more modest scenario views the worldwide energy utilization rate stabilizing at the present level of Eastern Europe: 5 kW per capita. This would lead to an overall rate of 65 TW in 2050, which is still too high. Finally, if the world average kept its present 2 kW per capita, the rate would grow to 26 TW by the middle of next century. Clearly, it is difficult to provide adequate energy for 11 billion people. This is one more reason for attempting to limit the planetary population growth.

The constant population increase has its Malthusian side. About 10% of the world's land area is used to raise crops—that is, it is **arable**

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<sup>†</sup> On 10/12/99, a 3.2 kg baby was born in Bosnia. Kofi Annan, General Secretary of the United Nations was on hand and displayed the new Bosnian citizen to the TV cameras because, somewhat arbitrarily, the baby was designated as the 6,000,000,000th inhabitant of this planet.

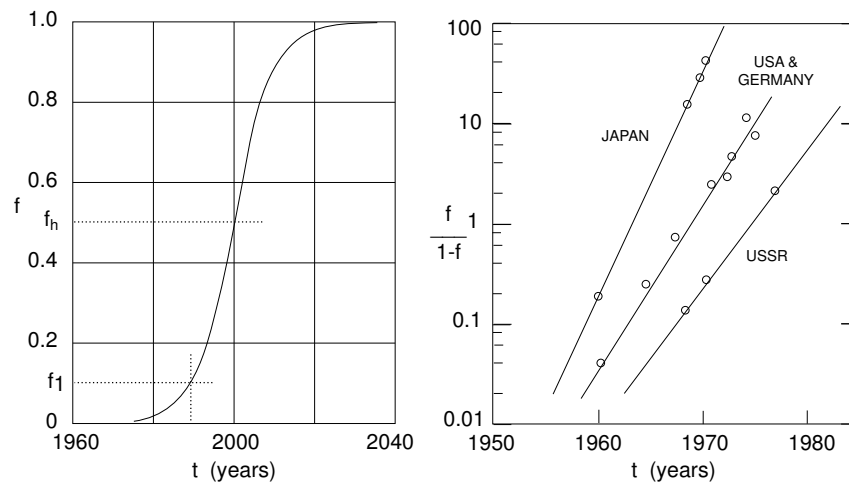


**land**, (See “Farming and Agricultural Technology: Agricultural Economics: Land, output, and yields.” Britannica Online.) This means that roughly 15 million km<sup>2</sup> or  $1.5 \times 10^9$  hectares are dedicated to agriculture. Up to the beginning of the 20th century, on average, each hectare was able to support 5 people (*Smil*), thus limiting the population to 7.4 billion people. More arable land can be found, but probably not enough to sustain 11 billion people. What limits agricultural productivity is nitrogen, one kilogram of which is (roughly) needed to produce one kilogram of protein. Although it is the major constituent of air, it is, in its elemental form, unavailable to plants and must either be “fixed” by appropriate micro-organisms or must be added as fertilizer.

Nitrogen fertilizers are produced almost exclusively from ammonia, and when used in adequate amounts can increase land productivity by nearly an order of magnitude. The present day and the future excess population of the planet can only exist if sufficient ammonia is produced. Although there is no dearth of raw materials for this fertilizer (it is made from air and water), its intensive use has a serious adverse environmental effect as discussed in the article by Smil.

## 1.7 The Market Penetration Function

A new technology, introduced in competition with an established one, may take over a progressively larger fraction of the market. Is it possible to forecast the rate at which such penetration occurs?



**Figure 1.5** Left: Typical penetration function vs time. Right: The penetration function of oxygen steel technology fits accurately the Fisher-Pry rule.

Let  $f$  be the fraction of the total market captured by the new technology. As time progresses,  $f$  grows from 0 to some value equal or less than 1. The latter corresponds to the new technology having totally replaced all competition. In due time,  $f$  may decrease again when a even newer technologies is introduced.

An empirical plot of the ascending phase of  $f$  vs time,  $t$ , has an “S” shape as exemplified by Figure 1.5 (left). A **market penetration time** is defined as  $\Delta T \equiv (t_h - t_1)$ , where  $t_h$  is the time at which  $f = 0.5 \equiv f_h$ , and  $t_1$  is the time at which  $f = 0.1 \equiv f_1$ .  $\Delta T$  may be negative if the technology in question is being replaced. It is then called the **abandonment time**. Fisher and Pry (1971) and Pry (1973) showed that when  $\ln \frac{f}{1-f}$  is plotted versus time, a straight line results. Figure 1.5 (right) illustrates an example of how the Fisher-Pry equation provides an excellent fit to the empirical data. The data show how, in four different countries, the use of oxygen in steel converters is gradually substituted for the older open-hearth and Bessemer technologies. The straight lines in the plots correspond to a regression of the type:

$$\ln \frac{f}{1-f} = at + b. \quad (1)$$

Constants  $a$  and  $b$  characterize the market and the particular technology considered. One would expect that the fractional rate of technology penetration of the market,  $\frac{1}{f} \frac{df}{dt}$ , is proportional to the fraction,  $(1-f)$ , of the market that has not yet been penetrated:

$$\frac{1}{f} \frac{df}{dt} = a(1-f). \quad (2)$$

The empirical evidence of Figure 1.5 (right) and of Equation 1 supports the model of Equation 2, because the former is the integral of the latter.

The quantities,  $a$  and  $b$  depend on the nature of the technology and on the specific location where the technology is being introduced. It is possible to generalize the Fisher-Pry equation by making it independent of these parameters.

For  $t = t_h$ ,  $f = 0.5$  and

$$\ln \frac{f}{1-f} = at_h + b = 0 \quad \therefore \quad b = -at_h. \quad (3)$$

For  $t = t_1$ ,  $f = 0.1$  and

$$\ln \frac{f}{1-f} = at_1 + b = -2.2. \quad (4)$$

Subtracting one equation from the other,

$$2.2 = a(t_h - t_1) = a\Delta t \quad \therefore \quad a = \frac{2.2}{\Delta t}. \quad (5)$$

Thus, the market penetration formula can be written as:

$$\ln \frac{f}{1-f} = 2.2 \frac{(t - t_h)}{\Delta t}. \quad (6)$$

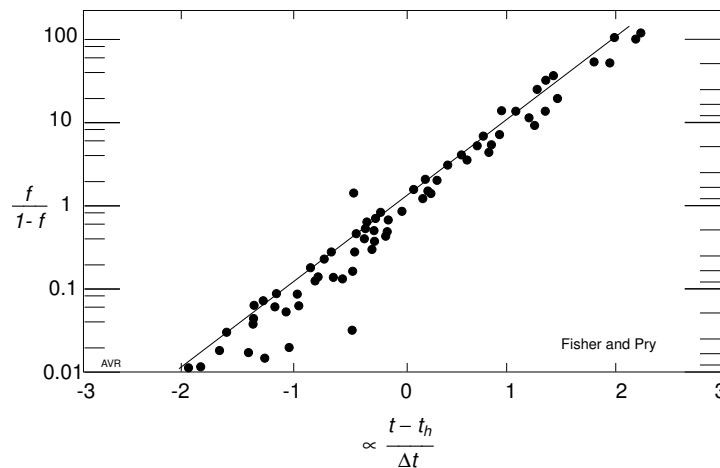
Equation 6 is a function of only the normalized independent variable,  $(t - t_h)/\Delta t$ . This permits presenting data with different  $a$ 's and  $b$ 's in a single graph. An example of such a plot is shown in Figure 1.6, prepared by Fisher and Pry. Data for 17 different cases of technology penetration are shown, with a surprisingly small scatter of points.

The Fisher-Pry model is insensitive to the overall market volume. Many factors that affect the market as a whole don't appear to influence its distribution among different technologies.

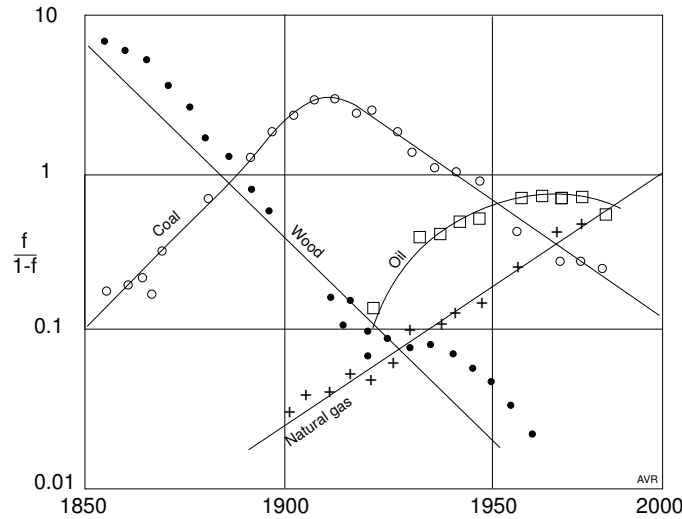
Figure 1.5 shows that the take over time for oxygen steel differed among countries: in Japan it was 5 years, in West Germany and in the USA 6, and in the Soviet Union 8 years. The rapid penetration of the technology was partially due to the fast depreciation of plants allowed by law.

Marchetti (1978) showed that the market penetration law is also applicable to energy. Figure 1.7 illustrates the fraction of the market supplied by a particular energy source as a function of time. The data are for the USA. The graph shows how energy from wood started abandoning the market in the 19th century owing to the introduction of coal as a source of fuel.

Coal, after penetrating the market for half a century, was forced out by oil and natural gas. Owing to the dispersed nature of the market, the time constants of both penetration and abandonment of energy products is much longer than that of most other technologies. Table 1.3 lists the different takeover times (abandonment times have a "minus" sign).



**Figure 1.6** Fisher-Pry plot for 17 different substitutions.



**Figure 1.7** Long before the OPEC intervention, the Fisher-Pry model would have predicted the current decline in oil consumption.

**Table 1.3**  
**Take-over and Abandonment Times.**

Wood	−38 years
Coal	38 years
Coal	−50 years
Oil	30 years

**Table 1.4**  
**Fisher-Pry Coefficients for Wood, Coal, and Gas.**

	<i>a</i>	<i>b</i>
Wood	−0.0585	110.20
Coal	−0.0439	85.18
Gas	0.0426	−84.64

Examine the period beginning in 1920. Wood, coal, and natural gas seem to have behaved according to the Fisher-Pry model. During this period, hydroelectric energy made a constant contribution of about 3.6% of the total. The regression coefficients for wood, coal and gas are shown in Table 1.4.

Since  $\sum f = 1$ , the fraction of the energy market supplied by oil can be calculated by subtracting from 1 the fractional contributions of the remaining fuels. When this is done, one arrives at the curve for oil penetration shown in Figure 1.7. It can be seen that it matches reasonably well the actual data (open squares).

The regression coefficients were obtained from data for 1920 through 1950 only; the rest of the information for these items resulted from extending the straight lines in the graph. Yet, the derived oil penetration curve

shows a decline starting around 1970, which, in fact, did occur. The recent decline in relative oil consumption could have been predicted back in 1950, years before the creation of OPEC! One can therefore conclude that the reduction in relative oil usage would have occurred regardless of the actions of OPEC. All OPEC did was to affect the overall price of energy.

## 1.8 Planetary Energy Resources

In Section 1.5, we pointed out that the rate of per capita energy utilization rose rapidly in the last century. This, combined with the fast increase in population mentioned in Section 1.6, leads one to the inescapable conclusion that we are facing a serious challenge if we hope to maintain these trends in the foreseeable future. To investigate what can be done to resolve this difficulty we must first inquire what energy resources are available (Section 1.8) and next (Section 1.9) how we are using the resources at present.

Figure 1.8 shows the planetary energy resources. These can be renewable or nonrenewable.

Geothermal energy has been used for a very long time in Iceland and more recently in Italy, New Zealand, and the United States. Great expansion of its contribution to the total energy supply does not seem probable.

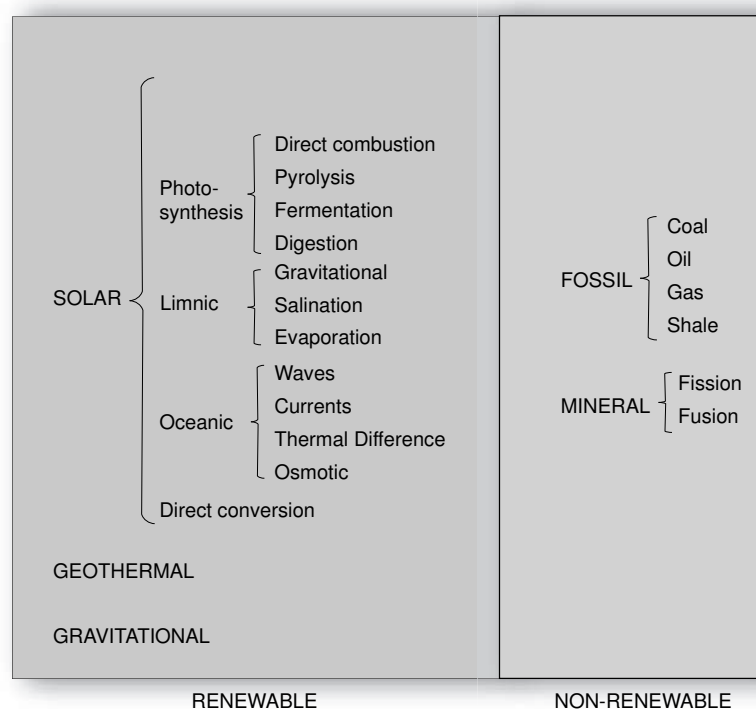
Gravitational energy—that is, energy from tides (see Chapter 16) has been used in France. Tides can only be harnessed in certain specific localities of which there is a limited number in the world.

Of the renewable resources, solar energy is by far the most abundant. A small part of it has been absorbed by plants and, over the eons, has been stored as coal, oil, and gas. Estimates of fossil reserves (as well as of nuclear fuel reserves) are extremely uncertain and are sure to be greatly underestimated because of incomplete prospecting. Table 1.5 gives us an idea of our fossil fuel reserves and Table 1.6 shows roughly estimated reserves of fissionable materials. These estimates do not include the old Soviet Union and China.

The values given in the table are very far from precise. They may, however, represent a *lower* limit. People who estimate these numbers tend to be conservative as testified by the fact that there is actually a secular *increase* in proved reserves. As an example, the proved reserves of dry natural gas, 2200 EJ in 1976, rose to 5500 EJ in 2002 notwithstanding the substantial consumption of gas in the intervening years.

For oil and gas, the table lists the sum of proved reserves, reserve growth and undiscovered reserves.

Proved reserves are fuels that have been discovered but not yet produced. Proved reserves for oil and gas are reported periodically in the *Oil and Gas Journal*.



**Figure 1.8** The energy resources of Earth.

**Table 1.5**  
**Known Fossil Fuel Reserves**

Methane clathrate	>100,000 EJ (1998)
Coal	39,000 EJ (2002)
Oil	18,900 EJ (2002)
Gas	15,700 EJ (2002)
Liquefied gas	2,300 EJ (2002)
Shale	16,000 EJ (?)

**Table 1.6**  
**Known Reserves of Fissionable Materials<sup>†</sup>**

$^{235}\text{U}$	2,600 EJ
$^{238}\text{U}$	320,000 EJ
$^{232}\text{Th}$	11,000 EJ

<sup>†</sup>Does not include the former USSR and China.

Reserve growth represents the increase in the reserves of existing fields owing to further development of these fields and to the introduction of better technology for their extraction.

Undiscovered reserves represent the best possible guess of possible new discoveries.

Reserve growths and undiscovered reserves are estimated by the US Geological Survey (<http://greenwood.cr.usgs.gov/energy/WorldEnergy/>)

DDS-60/>). For example, in 2002 the Oil and Gas Journal reported proved reserves of oil of 7280 EJ and the USGS estimated a growth of 4380 EJ and undiscovered oil reserves amounting to 5630 EJ adding up to the total of 18,900 EJ listed in the Table.

The indicated reserves also include 3000 EJ of proved dry natural gas that is currently too far from pipe lines to be economically transported to consumers.

In addition to the dry natural gas (mostly methane), a well will also produce other gases (propane, for example) that can be liquefied and shipped. The table lists a worldwide reserve of 2300 EJ in 2002.

For coal, the table shows only proved reserves. The total reserves for this fuel are, thus, substantially larger than listed.

One number in the table that is particularly uncertain is that referring to hydrated methane. William P. Dillon, a geologist of the USGS, testified in the U.S. House of Representatives in 1998, that “the amount of methane contained in the world’s gas hydrate accumulations is enormous, but estimates of the amounts are speculative and range over three orders-of-magnitude from about 100,000 to 270,000,000 trillion cubic feet [100,000 to 270,000,000 EJ] of gas.” We, being ultraconservative, listed the lower figure.

### Methane Clathrate

*Clathra* is the Latin word for “bar” or “cage”.

Atoms in a number of molecules group themselves in such a fashion that a cavity (or cage) is left in the center. The most famous of these arrangement is the “buckyball,” a molecule consisting of 60 carbon atoms arranged as a hollow sphere capable of engulfing a number of substances. Buckyballs, discovered in the early 1980s, are not alone among “hollow” molecules. Under appropriate circumstances, water will freeze forming a cage consisting, sometimes, of 20 water molecules, but more commonly, of 46 water molecules. The configuration is unstable (it decays into a common ice crystal) unless certain gases become trapped in the central cage of the large molecule. Gases commonly trapped are methane, ethane, propane, iso-butane, n-butane, nitrogen, carbon dioxide, and hydrogen sulfide.

The ice crystal consisting of 46 water molecules is able to trap up to 8 “guest” gas molecules (a water-to-gas ratio of 5.75:1). In natural deposits, methane is by far the most abundant and the one of greatest interest to the energy field. Usually, up to 96% of the cages are fully occupied. These solid hydrates are called **clathrates**.

(continues)

(continued)

The density of the clathrate is about  $900 \text{ kg/m}^3$ . This means that the methane is highly compressed. See Problem 1.28. Notwithstanding its low density, water ice clathrate does not float up from the bottom of the ocean because it is trapped beneath the ocean sediment.

Clathrates form at high pressure and low temperature under sea and are stable at sufficient depth. The methane is the result of anaerobic digestion of organic matter that continuously rains down on the ocean floor. See Chapter 13.

There is no mature technology for the recovery of methane from clathrates. Proposed processes all involve destabilizing the clathrate and include:

1. Raising the temperature of the deposits.
2. Depressurization the deposits.
3. Injecting methanol or other clathrate inhibitor.

The latter process may be environmentally undesirable.

There are dangers associated with methane clathrate extraction. The most obvious ones are the triggering of seafloor landslides and the accidental release of large volumes of methane into the Earth's atmosphere where it has a powerful greenhouse effect.

Read more about clathrates in *Clathrates: little known components of the global carbon cycle* <<http://ethomas.web.wesleyan.edu/ees123/clathrate.htm>>

## 1.9 Energy Utilization

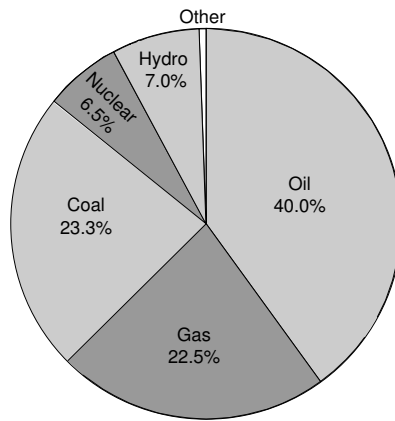
Most of the energy currently used in the world comes from non-renewable sources as shown in Figures 1.9 and 1.10, which display energy sources in 2001 for the whole world and for the United States, respectively. The great similarity between these two charts should not come as a surprise in view of the US using such a large fraction of the total world consumption.

What may be unexpected is that most of the renewable resources (geothermal, biomass, solar and wind) make such a small contribution to the overall energy picture. Figure 1.11 shows that as late as 1997 only 12% of the energy used to generate electricity in the USA came from renewable sources. Of these, 83% came from hydroelectrics. Thus, only 2% of the total came from the remaining renewables.

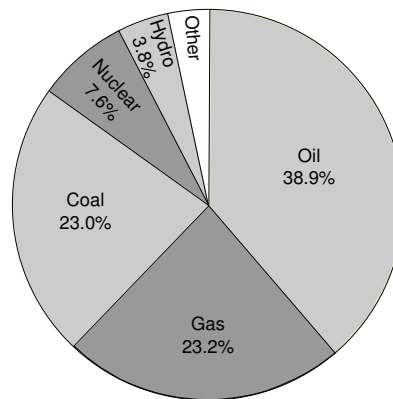
Disappointingly, so far, the contribution of solar and wind energy has been very small, much less than that of geothermal. Most of the renewable energy comes from hydro electric plants and some, from biomass.

1.16

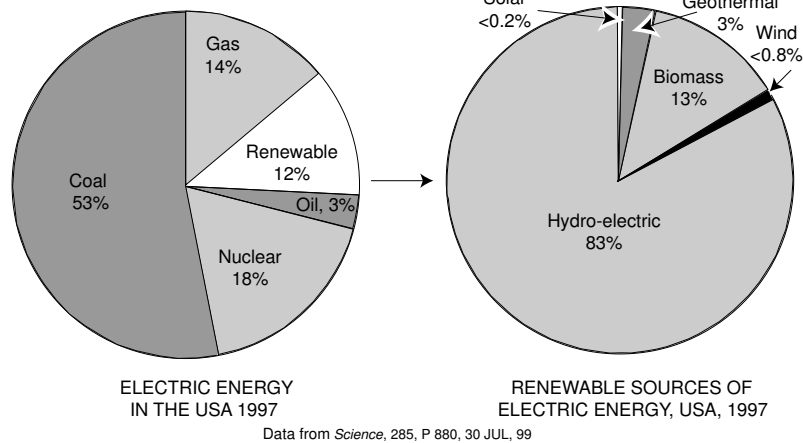




**Figure 1.9** Energy sources in the World.



**Figure 1.10** Energy sources in the US.

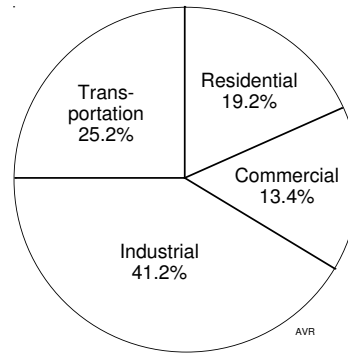


**Figure 1.11** Sources of electric energy in the United States.

**Table 1.7**  
Energy Use, USA 2001

Source	Used (EJ)	Capacity (GW)	Utilization factor
Thermal	9.69	600	51.1%
Nuclear	2.71	98	87.6%
Hydro	0.97	99	31.1%
Geothermal	0.26		
Wind	0.0208	4.28 <sup>†</sup>	15.4%
Other	0.015		

<sup>†</sup>This datum is from AWEA (American Wind Energy Association), all other are from EIA (Energy Information Administration.)



**Figure 1.12** The different users of energy in the USA.

For all sources of energy, the cost of the plant is proportional to the installed capacity, while the revenue is proportional to the energy generated. The **plant utilization factor** is the ratio of the energy produced to that which would be produced if the plant operated uninterruptedly at full capacity (Table 1.7).

Observe the extremely high utilization factor of nuclear plants and the rather small factor of wind generators, the latter resulting from the great variability of wind velocity. Although specific data for solar plants are not available, they also suffer from a low utilization factor owing the day/night cycle and the vagaries of meteorological conditions.

It is of interest to know which are the main users of energy in the United States.

American residences account for nearly 20% of all energy used. Most of it is used for ambient heating, an area in which considerable economy can be realized, especially through better home design.

Waste heat from electric power plants can be used for ambient heating in homes and offices. "District heating" is common in Sweden. Thermal power plants in that country, operate with an average 29% efficiency but 24% of the total fuel energy (from the heat rejected by the steam plant) is piped, as hot water, to buildings in the neighborhood. Thus, only 47% of the available combustion energy is wasted. In contrast, in the United States, a total of 68% of the combustion energy is wasted in spite of the larger average steam plant efficiency (32%). District heating requires the location of power plants in densely populated areas. This is, of course, inadvisable in the case of nuclear plants and large fossil fueled installations. However, fuel cell plants (see Chapter 9), being noiseless and pollution free, can be placed in a downtown area.

It is probably in the transportation sector (25% of the total energy use) where modern technology can have the most crucial impact. Fuel cell cars promise to increase automobile efficiency while reducing pollution.

## 1.10 The Ecology Question

We have shown that there is an almost unavoidable trend toward increasing energy utilization. We have also pointed out that at present the energy used is at least 85% of fossil origin. Finally, we have shown that the fossil fuel reserves seem ample to satisfy our needs for a good fraction of the next millennium. So, what is the problem?

Most of the easily accessible sources of oil and gas have already been tapped. What is left is getting progressively more expensive to extract. Thus, one part of the problem is economical. Another is political—most of the fuel used by developed nations is imported (using the large American reserves is unpopular and politicians hesitate to approve such exploration). This creates an undesirable vulnerability. There are also technological difficulties associated with the identification of new reserves and the extraction of fuels from more remote locations. The major obstacle, however, is ecological. Fossil fuels are still the most inexpensive and most convenient of all energy resources, but their use pollutes the environment, and we are quickly approaching a situation in which we can no longer dismiss the problem or postpone the solution.

By far, the most undesirable gas emitted is carbon dioxide whose progressively increasing concentration in the atmosphere (from 270 ppm in the late 1800 to some 365 ppm at present) constitutes a worrisome problem. It is sad to hear influential people (among them, some scientists) dismiss this problem as inconsequential, especially in view of the growing signs of a possible runaway ecological catastrophe. For instance, in the last few decades, the thickness of the north polar ice has decreased by 40% and on the first year of the current millennium, a summertime hole appeared in the polar ice. Since increased concentrations of CO<sub>2</sub> can lead to global warming, some people have proposed increasing the emission of SO<sub>2</sub> to stabilize the temperature because of the cooling effect of this gas. Even ignoring the vegetation-killing acid rain that would result, this proposal is equivalent to balancing a listing boat by piling stones on the other side.

The lack of public concern with the CO<sub>2</sub> problem may be due to the focus on planetary temperature rise. Although the growth in CO<sub>2</sub> concentration is very easily demonstrated, the conclusion that the temperature will rise, although plausible, is not easy to prove. There are mechanisms by which an increase of greenhouse gases would actually result in a *cooling* of Earth. For instance, increasing greenhouse gases would result in enhanced evaporation of the tropical oceans. The resulting moisture, after migrating toward the poles, would fall as snow thereby augmenting the albedo of the planet and, thus, reducing the amount of heat absorbed from the sun.

Some scientist and engineers who are less concerned with political correctness, are investigating techniques to reduce (or at least, to stabilize)

the concentration of atmospheric carbon dioxide. This can, in principle, be accomplished by reducing emissions or by disposing carbon dioxide in such a way as to avoid its release into the air. Emissions can be reduced by diminishing overall energy consumption (an utopian solution), by employing alternative energy sources, by increasing efficiency of energy use, and by switching to fuels that yield more energy per unit amount of carbon emitted. 1 kmole of methane,  $\text{CH}_4$ , when burned yielding liquid water and carbon dioxide, releases 889.6 MJ and emits 1 kilomole of carbon—it generates heat at a rate of 889.6 MJ per kilomole of carbon. n-heptane,  $\text{C}_7\text{H}_{16}$ , which can represent gasoline, releases 4820 MJ of heat per kilomole burned and emits 7 kilomoles of  $\text{CO}_2$ —a rate of 688.6 MJ per kilomole of carbon. Clearly, the larger the number of carbon atoms in the hydrocarbon molecule, the lower the ratio of the heat of combustion to the amount of carbon dioxide emitted because the ratio of hydrogen to carbon decreases. This is one reason for preferring methane to oil and oil to coal.

Alternative forms of energy are attractive but, at least for the present, are too expensive to seriously compete with fossil fuels.

In order to select a carbon dioxide disposal technique, it is important to inquire where nature stores the existing carbon.

Table 1.8 shows the estimated amount of carbon stored in different places.

Methods to dispose of  $\text{CO}_2$  could include:

### 1.10.1 Biological

Photosynthesis removes carbon dioxide from the air. The biomass produced must be preserved if it is to permanently affect the  $\text{CO}_2$  concentration. This means it cannot be burned or allowed to rot. There seems to be limited capacity for this method of  $\text{CO}_2$  disposal. It should be noted that the biological uptake rate of carbon is, at present, only  $0.002 \times 10^{15}$  kg year.

### 1.10.2 Mineral

$\text{CO}_2$  is removed naturally from the air by forming carbonates (principally of magnesium and calcium). The gas is removed by reacting with abundant silicates. However, this process is too slow to cope with man-made emissions.

**Table 1.8**  
**Stored Carbon on Earth.**

Oceans	45	$\times 10^{15}$ kg
Fossil fuels	10	$\times 10^{15}$ kg
Organic matter	2.4	$\times 10^{15}$ kg
Atmosphere	0.825	$\times 10^{15}$ kg

1.20

### Carbon in the atmosphere

How much carbon is there in the atmosphere?

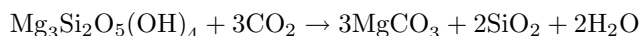
The surface area of earth is  $510 \times 10^{12} \text{ m}^2$ , while the scale height of the atmosphere is around 8800 m (see the section on Boltzmann's law in Chapter 2). Consequently the volume of air (all of it compressed to 1 atmosphere pressure) is  $510 \times 10^{12} \times 8800 = 4.5 \times 10^{18} \text{ m}^3$ .

Present day atmospheric  $\text{CO}_2$  concentration is  $13.5 \times 10^{-6} \text{ kmol/m}^3$ . Thus, the atmosphere contains  $13.5 \times 10^{-6} \times 4.5 \times 10^{18} = 61 \times 10^{12} \text{ kmol}$  of  $\text{CO}_2$  and, therefore,  $61 \times 10^{12} \text{ kmol}$  of carbon. Since the atomic mass of carbon is 12 daltons, the mass of carbon in the atmosphere is  $0.73 \times 10^{15} \text{ kg}$ . Compare with the  $0.825 \times 10^{15} \text{ kg}$  of the table.

A simpler way to achieve about the same result is to consider that the atmospheric pressure at sea level is  $1 \text{ kg/cm}^2$  or  $10^4 \text{ kg/m}^2$ . Consequently, the total mass of the atmosphere is  $510 \times 10^{12} \times 10^4 = 510 \times 10^{16}$ . Of this  $360 \times 10^{-6}$  is carbon dioxide and  $12/44$  of this is carbon. Carbon content of the atmosphere is  $510 \times 10^{16} \times 365 \times 10^{-6} \times 12/44 = 0.51 \times 10^{15} \text{ kg}$ , a result comparable with the previous one.

Ziock et al. propose the use magnesium silicates to sequester carbon dioxide at the point where fossil fuels are burned. Enormous deposits of magnesium oxide-rich silicates exist in the form of olivines and serpentines.

For serpentine, the net reaction involved is



Notice that the end products are materials that already exist naturally in great abundance.

Substantial additional research is needed to improve the proposed disposal system and to make it economical.

#### 1.10.3 Subterranean

$\text{CO}_2$  can be sequestered underground as the oil industry has been doing (for secondary oil recovery) for more than 50 years. The volume of the exhaust gases of a combustion engine is too large to be economically stored away. It is necessary to separate out the carbon dioxide, a task that is not easy to accomplish. One solution is proposed by Clean Energy Systems, Inc. of Sacramento, CA. The suggested equipment extracts oxygen from air (a well developed process) and mixes this gas with the fuel. Combustion produces steam and  $\text{CO}_2$  at high temperature and pressure and drives several turbines at progressively lower temperatures. The water in the final exhaust is condensed and recycled leaving the carbon dioxide

to be pumped, at 200 atmospheres, into an injection well. At present, no turbines exist capable of operating at the high temperature (over 3000 C) of the combustion products. See *Anderson, et al.*

#### 1.10.4 Undersea

The Norwegian government imposes a stiff carbon dioxide emission tax that has made it economical to install disposal systems. They pump the gas deep into the ocean. It appears that liquid carbon dioxide can be injected into the seas at great depth and that it will stay there for a long time. More work is required to see if such scheme is indeed feasible and economical.

### 1.11 Nuclear Energy

Chemical fuels, such as oil or methane, release energy when the atoms in their molecules are rearranged into lower energy configurations. The energies involved are those of molecular binding and are of the order of some tens of MJ/kmol. When the *components* of an atom are arranged into lower energy configurations, then the energy released is orders of magnitude larger (GJ/kmole) because of the much larger intra-atomic binding energies.

The internal structure of atoms can be changed in different ways:

1. An atomic nucleus can be bombarded with a neutron, absorbing it. A different atom emerges.
2. An atom can spontaneously change by emitting either electrons (beta-rays) or helium nuclei (alpha-rays). Such radioactive decay releases energy which can be harvested as, for instance, it is done in **Radioisotope Thermal Generators** (RTG).
3. Atoms with large atomic number can be made to break up into smaller atoms with the release of energy. This is called **nuclear fission** and requires that the atomic number,  $Z$ , be larger than 26.
4. Atoms with low atomic numbers can be assembled into a heavier one, releasing energy. This is called **nuclear fusion** and requires that the final product have an atomic number smaller than 26.<sup>†</sup>

Nuclear energy has developed a bad reputation especially after the Chernobyl accident. Nevertheless it is still a source of substantial amounts of energy in many countries. In 2001, the US led the world in installed capacity—98 GW, followed by France (60 GW) and Japan (42 GW).<sup>††</sup>

The utilization factor of nuclear plants was excellent. In the US, the plants generated 87.6% of the energy they would have delivered if they had operated uninterruptedly at full power. In France, this number was 69.5% and in Japan, 75.4%.<sup>††</sup>

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<sup>†</sup> All are transmutations, the age-old dream of medieval alchemists.

<sup>††</sup> The French and the Japanese data are for 1996.

Of the total electricity generated, nuclear plants in the US contributed a relatively modest 18%, while in France, heavily reliant on this form of energy, the contribution was 76.1%. In Japan, it was 33.4%. In 2000, Germany decided to phase out its 19 nuclear power plants. Each one was assigned a 32-year life after which they would be deactivated. Many plants have already operated more than half of their allotted life time.

The cost of nuclear electricity is high, about double of that from fossil fuel. In the US (1996) it was 7 cents/kWh, while that of a state of the art natural gas plant it was 3 cents/kWh (Sweet, William #1). Advanced reactor designs may bring these costs down considerably while insuring a greater safety in the operation of the plants (Sweet, William #2). This promised reduced cost combined with the ecological advantage of no greenhouse gas emission—a growing concern—may lead to renewed popularity of nuclear generators.

The major objection to fission type reactors is not so much the danger of the operation of the power plants, but rather the problem of disposing of large amounts of long-lived radioactive by-products. If the need for such disposal can be avoided, then there is good reason to reconsider fission generators as an important contributor to the energy supply system.

Specifications of new generation nuclear fission reactors might include (not necessarily in order of priority), the following items:

1. Safety of operation (including resistance to terrorist attacks)
2. Affordability
3. Reliability
4. Absence of weaponizable sub-products
5. Absence of long-lived waste products
6. Ability to transmute long-lived radioactive waste products from old reactors into short-lived radioactive products

The U.S. Department of Energy is funding research (2004) in several technologies that might realize most of the specifications above. One of these is the **heavy metal nuclear reactor** technology. Although the technology is complicated, it appears that this type of reactor may be able to not only produce wastes with relatively short half-lives (100 years contrasted with 100,000 years of the current waste), but in addition may be able to use current type waste as fuel thus greatly alleviating the waste disposal problem. Furthermore, because heavy-metal reactors operate at high temperatures (yet at low pressures), the thermolytic production of hydrogen (see Chapter 10) for use in fuel cell-driven automobiles looms as a good possibility. For further reading on this topic see Loewen.

The waste disposal problem is absent in fusion devices. Unfortunately, it has been impossible to demonstrate a working prototype of a fusion machine, even after several decades of concerted research.

**Table 1.9**  
**Masses of Some Particles Important to Nuclear Energy**

Particle	Symbol	Mass (daltons <sup>†</sup> )	Mass (kg)
electron ♣	$e$	0.00054579903	$9.1093897 \times 10^{-31}$
muon •	$\mu$	0.1134381	$1.883566 \times 10^{-28}$
proton ♣	$p$	1.007276467	$1.672648 \times 10^{-27}$
neutron ♣	$n$	1.008664909	$1.6749286 \times 10^{-27}$
${}^1_1\text{H}$		1.007825032	$1.673533967 \times 10^{-27}$
${}^2_1\text{D}$		2.014101778	$3.344496942 \times 10^{-27}$
${}^3_1\text{T}$		3.016049278	$5.008271031 \times 10^{-27}$
${}^3_2\text{He}$		3.016029319	$5.008237888 \times 10^{-27}$
${}^4_2\text{He}$		4.002603254	$6.646483555 \times 10^{-27}$
alpha ♣	$\alpha$	4.001506175	$6.644661810 \times 10^{-27}$
${}^5_3\text{Li}$		5.01254	$8.323524107 \times 10^{-27}$
${}^6_3\text{Li}$		6.015122794	$9.988353127 \times 10^{-27}$
${}^7_3\text{Li}$		7.01600455	$1.165035751 \times 10^{-26}$
${}^{10}_5\text{B}$		10.012937	$1.662688428 \times 10^{-26}$
${}^{11}_5\text{B}$		11.009305	$1.82814 \times 10^{-26}$

<sup>†</sup>The dalton is not yet the official name for the atomic mass unit.

To do even a superficial analysis of the technical aspects of nuclear reactions, we need to know the masses of some of the atoms involved. See table above. Most of mass values are from Richard B. Firestone. Those marked with a ♣ are from Audi and Wapstra (1993), and the one marked with a • is from a different source. It can be seen that the precision of the numbers is very large. This is necessary because, in calculating the energy released in a nuclear reaction, one uses the small difference between large numbers which is, of course, extremely sensitive to uncertainties in the latter.

The listed values for the masses of the nucleons (the proton and the alpha, in the table) are nearly the values of the masses of the corresponding atoms minus the mass the electron(s). On the other hand, there is a large difference between the the mass of a nucleon and the sum of the masses of the component protons and neutrons. Indeed, for the case of the alpha, the sum of the two protons and the two neutrons ( 4.03188278 daltons) exceeds the mass of the alpha (4.001506175 daltons) by 0.030376606 daltons—about 28 MeV of mass. This is, of course, the large **nuclear binding energy** necessary to overcome the great electrostatic repulsion between the protons.

1.24



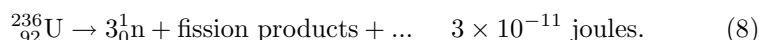
### 1.11.1 Fission

There are at least three fissionable elements of practical importance:  $^{235}\text{U}$ ,  $^{239}\text{Pu}$  and  $^{233}\text{U}$ . Of these, only  $^{235}\text{U}$  is found in nature in usable quantities;  $^{239}\text{Pu}$  and  $^{233}\text{U}$  must be created by transmutation from “fertile” materials, respectively  $^{238}\text{U}$  and  $^{232}\text{Th}$ .

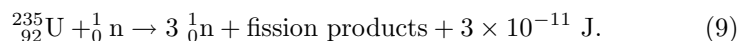
A nuclear fission reaction (with a corresponding release of energy) occurs when a fissionable material interacts with thermal, i.e., low energy, neutrons. The collision of high energy neutrons with  $^{235}\text{U}$ , for example, is elastic, whereas low energy neutrons are captured:



The resulting  $^{236}\text{U}$  decays with the emission of alpha-particles (lifetime 7.5 seconds). More importantly, the uranium also suffers spontaneous fission:



Thus, under the proper circumstances,  $^{235}\text{U}$  absorbs a neutron and the resulting atom splits into smaller nuclei simultaneously releasing 3 neutrons and about  $3 \times 10^{-11}$  joules of energy:



Per kilogram of  $^{235}\text{U}$ , the energy released is

$$\frac{3 \times 10^{-11} \frac{\text{J}}{\text{atom}} \times 6 \times 10^{26} \frac{\text{atoms}}{\text{kmol}}}{235 \frac{\text{kg}}{\text{kmol}}} = 77 \quad \text{TJ/kg}.$$

Compare this with the energy from chemical reactions which is frequently of the order of a few tens of MJ/kg.

When Otto Hahn, in 1939, demonstrated uranium fission, it became immediately obvious that a sustained “chain” reaction would be achievable. To such an end, all that was needed was to use one of the emitted neutrons to split a new uranium atom. In trying to build such a fission reactor, a number of problems had to be overcome.

1. The  $^{235}\text{U} + {}^1_0\text{n}$  reaction requires slow (thermal) neutrons. The high energy neutrons emitted will not do. Thus, these neutrons must be made to transit through some material that has the property of slowing the particle down without absorbing it. Examples of such “moderating” substances are heavy water and graphite.

2. Fast neutrons may be absorbed by impurities in the fuel or in the moderator. The fuel is a mixture of  $^{235}_{92}\text{U}$  and  $^{238}_{92}\text{U}$ . The latter is an abundant “impurity” that absorbs fast neutrons but not slow ones. To reduce neutron losses, it may be necessary to “enrich” the fuel, i.e., increase the  $^{235}_{92}\text{U}/^{238}_{92}\text{U}$  ratio.<sup>†</sup> It is also necessary to place the fuel into a number of long rods embedded in a mass of moderator. This configuration allows most of the fast neutrons to escape the fuel region and reach the moderator where they are slowed and may eventually reenter one of the fuel rods. They now have insufficient energy to interact with the  $^{238}_{92}\text{U}$  but will do so with  $^{235}_{92}\text{U}$  perpetuating the reaction.

Clearly, it is essential that exactly one of the released neutrons is, on average, used to trigger a new fission. If more than one, the reaction will grow exponentially, if less, it will die out. Control systems are used to adjust this number to precisely one. Fortunately, the process is, to a degree, self adjusting—if the reaction rate rises, so will the temperature, and this reduces the probability of neutron capture.

Uranium isotopes cover the range from 227 to 240 in atomic mass, but natural uranium consists chiefly of:

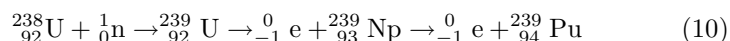
**Table 1.10**  
**Uranium Isotopes**

Isotope	Abundance (%)	Lifetime (years)
$^{238}\text{U}$	99.283	$4.5 \times 10^9$
$^{235}\text{U}$	0.711	$7.1 \times 10^8$
$^{234}\text{U}$	0.005	$2.5 \times 10^5$

It is estimated that in the Western World there are reserves of uranium oxide ( $\text{U}_3\text{O}_8$ ) amounting to some  $6 \times 10^9$  kg, but only  $34 \times 10^6$  kg are fissionable, corresponding to an available energy of 2600 EJ. Compare this with the 40,000 EJ of available coal energy.

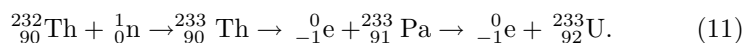
The relatively modest resources in fissionable uranium led to “breeder reactors” in which fertile materials are transformed into fissionable ones.

Take  $^{238}\text{U}$ , which suffers inelastic collisions with high energy neutrons (neutrons from fission):



<sup>†</sup> No enrichment is needed if the moderator is heavy water ( $\text{D}_2\text{O}$ ) as used in the CANDU reactor. This is the **CAN**adian **D**euterium **U**ranium, pressurized heavy water reactor that uses natural (unenriched) uranium and heavy water as both moderator and coolant.

or take  $^{232}\text{Th}$ :



By creating plutonium in a breeder reactor, all uranium can be made to yield energy: 320,000 EJ become available. Even larger amounts of energy could be derived from thorium.

### 1.11.2 Fusion

The main objections to fission reactors are related to

1. lack of safety,
2. dangerously radioactive “ashes,” and
3. scarcity of fuel.

Fusion reactors may overcome all of the above objections.

The reaction that is, by far, the easiest to ignite is<sup>†</sup>



To estimate the reaction energy released, one calculates the amount of mass lost. The mass of  ${}^3_1\text{T}$  is  $5.00827 \times 10^{-27}$  kg as given by the table at the beginning of this subsection. The mass of the deuterium is  $3.3434 \times 10^{-27}$  kg so that the mass of the left side of the equation is  $8.3526 \times 10^{-27}$  kg. On the right-hand side of the equation the sum of the masses of the alpha particle (the helium ion) and the neutron is  $8.3214 \times 10^{-27}$  kg, a deficit of  $3.12 \times 10^{-29}$  kg. When multiplied by  $c^2$ , this yields an energy of  $2.80 \times 10^{-12}$  joules per deuterium/tritium pair. The correct value is very slightly larger (it is nearer  $2.81 \times 10^{-12}$  J). The small discrepancy is mainly due to the fact that we used the mass of the atoms instead of that of the corresponding ions. The reaction yields 337 TJ per kg of tritium/deuterium alloy or 562 TJ per kg of tritium.

The energy released by the reaction is carried by both the alphas and the neutrons. The conversion of the neutron energy to usable forms has an efficiency of only some 40% because the particles are uncharged and heat management and mechanical heat engines are involved. On the other hand, the alphas can be directly converted to electricity at a much higher efficiency ( $\approx 90\%$ ). See Rostoker, Monkhurst, and Binderbauer (1997); Moir and Barr (1973); Momota et al. (1998); Yoshikawa et al. (1991); Bloch and Jeffries (1950). In addition, the heavy neutron flux creates serious radioactivity and material destruction problems. Consequently, it is

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<sup>†</sup> The larger the atomic number,  $Z$ , the greater the difficulty of reaction owing to the large electrostatic repulsion between nuclei.

important to know how the released energy is divided between the alphas and the neutrons. This can be done by assuming that the momenta are equally divided between the two types of particles:

$$m_{\alpha}v_{\alpha} = m_nv_n, \quad (13)$$

and combining this with the energy equation,

$$\frac{1}{2}m_{\alpha}v_{\alpha}^2 + \frac{1}{2}m_nv_n^2 = W = W_{\alpha} + W_n. \quad (14)$$

Here,  $m_{\alpha}$  is the mass of the alpha,  $m_n$  is the mass of the neutron,  $v_{\alpha}$  is the velocity of the alpha,  $v_n$  is the velocity of the neutron, and  $W$  is the energy released by one pair of reacting atoms. Solving these simultaneous equations leads to

$$W_{\alpha} = \frac{W}{\frac{m_{\alpha}}{m_n} + 1} \quad (15)$$

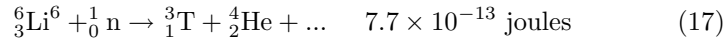
and

$$W_n = \frac{W}{\frac{m_n}{m_{\alpha}} + 1}. \quad (16)$$

For the reaction under consideration, it is found that neutrons carry about 14 MeV, while the more massive alphas carry only some 3.5 MeV.

The T+D reaction is popular because of its high reactivity, which should facilitate ignition, and because the atomic number of the fuel is  $Z = 1$ , thus minimizing radiation losses. This is because radiation is a function of  $Z^2$ . However, it has drawbacks:

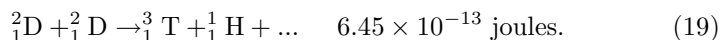
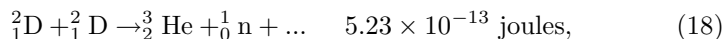
1. One neutron is emitted for each  $2.8 \times 10^{-12}$  J generated, whereas, in fission, the rate is one neutron per  $10^{-11}$  J. Thus, the neutron bombardment is serious: it radioactivates substances and weakens structures by causing dislocations in the crystal lattice and by generating hydrogen bubbles inside materials.
2. As pointed out before, most of the energy is in the neutron stream reducing the recovery efficiency.
3. Although deuterium is not radioactive, tritium is radioactive with a lifetime of 12 years. It has the tendency to stick around by replacing normal hydrogen in water molecules.
4. There is no natural source of tritium; it must be obtained from lithium:



Thus, each lithium atom yields  $2.8 \times 10^{-12} + 7.7 \times 10^{-13} = 3.57 \times 10^{-12}$  J. One kg of lithium yields 350 TJ.

The world reserves of lithium are not known accurately. Conservative estimates are of  $10^{10}$  kg. However, most of this is  ${}^7\text{Li}$ . The desired isotope,  ${}^6\text{Li}$ , has a relative abundance of 7.4%. Consequently, one can count on only  $740 \times 10^6$  kg of this material or 260,000 EJ of energy.

In order of ease of ignition, the next two reactions are



The above reactions have equal probability of occurring.

The tritium produced will react with the deuterium according to Reaction 12. The average energy of the D+D reaction is

$$\frac{(5.1 + 6.40 + 28.0) \times 10^{-13}}{5} = 7.9 \times 10^{-13} \text{ J per D atom}. \quad (20)$$

The D+D reaction is still dirty (neutronwise) and still involves a radioactive gas (tritium). However, it does not use a fuel of limited abundance, such as lithium. It uses only deuterium, which is available in almost unlimited amounts. In common water, there is one  $\text{D}_2\text{O}$  molecule for every 6700  $\text{H}_2\text{O}$  molecules. One can estimate roughly how much deuterium is available:

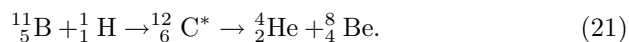
The oceans cover about 2/3 of the Earth's surface, which is  $5.1 \times 10^{14} \text{ m}^2$ . Assuming an average depth of 3000 m, the ocean has a volume of  $10^{18} \text{ m}^3$  and a mass of  $10^{21} \text{ kg}$ . Of this, 1/9 is the mass of hydrogen, and 2/6700 of the latter is the mass of deuterium, amounting to some  $3.3 \times 10^{16} \text{ kg}$  or about  $10^{31} \text{ J}$ —an amount of energy that, for practical purposes, can be considered unlimited.

Next, in order of ignition difficulty is the  ${}^2\text{D} + {}^3\text{He}$  reaction that burns cleanly: no radioactive substances are involved and no neutrons are generated. Also clean is the  ${}^3\text{H} + {}^3\text{He}$  reaction.

The catch in these reactions is that there is no natural  ${}^3\text{He}$  on earth; it must be made from the (dirty) fusion of Li and H. However, it is estimated that over a billion tons of the material exists on the moon. This may, one day, justify a mining operation on our satellite.

The  ${}^3\text{H}$  on the moon comes from the solar wind that has, for billions of years, deposited it there. The  ${}^3\text{H}$  on earth is trapped by the atmosphere and is eventually evaporated away.

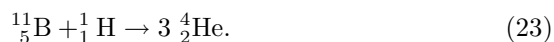
An interesting reaction involves  ${}^{11}\text{B}$ , the common isotope of boron:



${}^{12}_6\text{C}^*$  is nuclearly excited carbon which spontaneously decays into an alpha and  ${}^8_4\text{Be}$  a very unstable atom with a lifetime of  $2 \times 10^{-16}$  seconds. Fortunately, it is an alpha-emitter:



The overall reaction is:



or, using a different notation



It appears that this **triple alpha** reaction can be made to sustain itself in a **colliding beam fusion reactor** (See Rostoker, Binderbauer, and Monkhorst, 1997) but this has not yet been demonstrated. If it does work, we would have a clean fusion reactor using abundantly available fuel and capable of operating in units of moderate size, in contrast with the T+D reaction in a Tokamak with must be 10 GW or more if it can be made to work at all.

It should be noticed that  ${}^{10}\text{B}$  will also yield a triple-alpha reaction when combining with a deuteron:



Both isotopes of boron considered above are abundant, stable, and nonradioactive. Natural boron consists essentially of 20%  ${}^{10}\text{B}$  and 80%  ${}^{11}\text{B}$ .

The triple alpha reaction may also be an important player in the cold fusion process (if such process exists at all). See the next subsection.

Table 1.11 lists the percentage of the energy of a reaction that is carried away by neutrons.

Although fusion reactors have not yet been demonstrated<sup>†</sup>, there is a possibility that they will become the main source of energy some 50 years from now. If so, they may provide the bulk of the energy needed by humanity and the energy crunch will be over.

**Table 1.11**  
**Neutron Yields**

Reaction	% of energy Carried by Neutrons
D + T	65...75
D + D	20...45
B + H	< 0.1

<sup>†</sup> Fusion research dates back to at least 1938 when Jacobs and Kantrowitz built a first magnetic confinement fusion reactor at NACA's (now NASA) Langley Memorial Aeronautical Laboratory.

### 1.11.3 Cold Fusion

At the beginning of the millennium, when this subsection was being rewritten, the cold fusion question remained unresolved. So far, no one has been able to reproduce the claims of Pons and Fleishmann, but, on the other hand, no one has been able to disprove the existence of cold fusion. As a matter of fact, cold fusion can and has been demonstrated. Let us review what we know for sure of this topic.

As indicated in Subsection 1.11.2, deuteron will react spontaneously with deuteron in one of these two reactions:



These reactions have about the same probability of occurrence and they produce a substantial amount of energy. The problem is that the probability of occurrence (under normal conditions) is extremely small, of the order of one fusion per galaxy per century according to a good humored scientist,

It is easy to understand the reluctance of the  ${}^2\text{D}$  atoms to get together: they carry positive charges and therefore repel one another. This can be overcome by imparting sufficient kinetic energy to the atoms, as, for instance, by heating them to extreme temperatures as in thermonuclear fusion.

There is a neat trick suggested by Alvarez (late professor of the University of California at Berkeley and Nobel Prize winner) that increases by 85 orders of magnitude the reaction cross-section (read probability). Replacing the orbital electron of the deuterium by a muon which is 207 times heavier, collapses the orbital by a large factor.<sup>†</sup> Muon mediated fusion can be observed in the laboratory as Jones (Brigham Young) demonstrated. The catch is that it takes more energy to create the muon than what one gets from the fusion.

Thus, cold fusion certainly does occur. More than that, cold fusion occurs (almost certainly) even when not mediated by muons.

Jones described an experiment that appears to prove just that. He used an electrolytic cell consisting of a platinum positive electrode and a palladium (sometimes, titanium) negative electrode. The electrolyte was  $\text{D}_2\text{O}$  (heavy water). Since water is a poor conductor of electricity, salts had to be added to the solution. Here is Jones's extraordinary recipe:

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<sup>†</sup>A deuteron/electron molecule is about 74,000 fm in size, while the deuteron/muon molecule is only 250 fm in size. The nuclei are, therefore, 300 times closer together and this raises enormously the probability of fusion.

“The electrolyte is a mixture of about 160 g of deuterium oxide ( $D_2O$ ) plus various metal salts in about 0.2 g amounts each:  $FeSO_4$ ,  $NiCl_2$ ,  $PdCl_2$ ,  $CaCO_3$ ,  $LiSO_4$ ,  $NaSO_4$ ,  $CaH_4(PO_4)_2$ ,  $TiOSO_4$ , and a very small amount of  $AuCN$ .”

A chemist might be horrified by the cocktail above—it would be hard to tell what is going on.<sup>†</sup>

When a current was forced through the cell, a small flux of neutrons with a characteristic energy of 2.5 Mev was observed. Jones, a physicist, did a good job of neutron detection. Since 2.5 Mev is the energy of the neutrons in Reaction 27, this experiment tends to show that indeed fusion is going on.

Jones observed that some 8 hours after start of operation, the neutron “signal” turned off by itself. This effect was attributed to the poisoning of the palladium electrode by deposition of metals from the solution. In fact, etching the electrode revived the cell.

The reaction rate observed by Jones was small, perhaps  $10^{-20}$  fusions per deuterium pair per second. This could be explained if the deuterium molecules were somehow squeezed from 74,000 fm to half this distance by their residence in the palladium lattice.<sup>††</sup> Jones dubs this **piezonuclear fusion**.

Pons and Fleishmann ran similar experiments but, being chemists not physicists, adopted a simpler electrolyte: a  $LiOH$  solution in  $D_2O$  (heavy water). They also failed to make careful neutron measurements. What they reported is that, after a prolonged pre-cooking, some cells suddenly developed a great deal of heat, billions of times more than in the Jones experiment. Unfortunately, these results were never reproduced by other experimenters and this casts severe doubts on their validity. Here is where I will don my devil’s advocate mantle and, just for the fun of it, will defend the P&F results.

In a lecture delivered at the Utah University on March 31, 1989, Stanley Pons relates the most spectacular of his results. “A cube of palladium with a volume of  $1\text{ cm}^3$  was used as cathode of an electrolyzer with lithium hydroxide dissolved in  $D_2O$  as an electrolyte. A current of  $250\text{ mA/cm}^2$  was applied for several weeks/months [sic] with nothing remarkable happening. A Geiger counter detected no radiation. The current was cut to  $125\text{ mA/cm}^2$  late one day, and next morning the cube of palladium and the electrolysis cell were gone. A nearby Geiger counter was also ruined.”<sup>†††</sup>

<sup>†</sup> Jones was trying to create a chemical environment somewhat like the one in the soil because he was trying to show that some of the internal heat of our planet is generated by deuterium fusion.

<sup>††</sup> A possible cause of the squeezing would be the increase of the electron mass to a few times its free mass.

<sup>†††</sup> As related by Patrick Nolan, 1989 (paraphrased).



There was a long delay (several days, at least) before heat evolved. Since the Jones cell poisons itself in 8 hours, this cell will never reach the primed state and no heat can be observed.

Why such a delay? Hydride hydrogen storage systems (see Chapter 11) are well known and are commercially available. One popular system uses a TiFe alloy to absorb  $H_2$ . Many other metals and alloys will do the same. Palladium, in particular, is a notorious  $H_2$  absorber. It is not used commercially owing to its high price.

When TiFe powder (after duly activated) is exposed to hydrogen, it will form a (reversible) hydride, TiFeH. If the amount of hydrogen is small, there will be a mixture of TiFe and TiFeH in the powder. This mixture, called  $\beta$ -phase, has the empirical formula  $TiFeH_x$ , where  $x$  becomes 1 when all the material has been hydrided.

After full hydridization, addition of more hydrogen will cause the formation of a di-hydride,  $TiFeH_2$ , ( $\gamma$ -phase). Clearly, the hydrogen is more densely packed in the (di-hydride)  $\gamma$ -phase than in the  $\beta$ -phase. It is, therefore, plausible that the fusion will proceed faster once the  $\gamma$ -phase is reached. How long does it take to reach this  $\gamma$ -phase?

In the described experiment, Pons used a current density of  $250 \text{ mA/cm}^{-2}$ , a total current of 0.042 A. This corresponds to a production of  $2.6 \times 10^{17}$  deuterons/second. Each cubic centimeter of palladium contains  $68 \times 10^{21}$  atoms. Thus, it takes 260,000 seconds or some 72 hours (3 days) for the palladium, in this particular experiment, to start becoming di-hydrided. This assumes that all the deuterons produced are absorbed by the palladium and, thus, the time calculated is a rough lower limit.

Could the heat have resulted from a chemical reaction? The highest enthalpy of formation of any palladium salt seems to be 706 MJ/kmole, for palladium hydroxide. Atomic mass of palladium is 106 daltons and density is  $12 \text{ g cm}^{-3}$ . This means that one gets  $80 \text{ kJ cm}^{-3}$  chemically. Pons and Fleischmann have (they say) gotten  $5 \text{ MJ cm}^{-3}$ , two orders of magnitude more than chemistry allows.

Conclusion:

1. The heat produced cannot be due to classical fusion reaction (insufficient neutrons, tritium, and  $\gamma$ -rays).
2. The heat produced cannot be due to chemical reaction.
3. Then, simplistically, the heat was not produced.

There is at least one more possible reaction which occurs very rarely:



As written above, this reaction cannot take place because two particles are converted into a single particle and it is impossible to conserve simulta-

neously energy and momentum under such conditions. For the reaction to proceed, it is necessary to shed energy and, in classical physics, this is done by emitting a 16 MeV  $\gamma$ -ray. Pons did not report  $\gamma$ -rays. There is still an outside possibility that the energy can be shed by some other mechanism such as a phonon, although physicists tell me that this is nonsense. Observe that Reaction 28 produces one order of magnitude more energy per fusion than do Reactions 26 and 27.

So far, we have attempted to explain the hypothetical cold fusion as the result of deuteron-deuteron reaction. It has been difficult to account for the absence of the expected large fluxes of neutrons or gamma rays. It is even more difficult to imagine such reaction proceeding when common water is used in place of heavy water. Nevertheless, some experimentalists make exactly such a claim.

There have been suggestions that cold fusion actually involves nuclear reactions other than those considered so far. Let us recapitulate what has been said about cold fusion.

1. The results, if any, are not easily reproduced.
2. No substantial neutron flux has been detected. This seems to eliminate the deuteron-deuteron reactions of Equations 26 and 27.
3. No substantial gamma ray flux has been detected. This eliminates the classical form of the deuteron-deuteron reaction of Equation 28.
4. Reactions are reported to be highly dependent on the exact nature of the palladium electrode.
5. Reactions have been reported with a  $\text{H}_2\text{O}$  instead a  $\text{D}_2\text{O}$  electrolyte.

The following cold fusion mechanism fitting the above observations has been recently proposed.

Boron is a common impurity in palladium. Natural boron exist in the form of two isotopes with the relative abundance of 20% for  $^{10}\text{B}$  and 80% for  $^{11}\text{B}$ . Thus, under some special circumstances, the two triple-alpha reactions of Equations 24 and 25 might occur. They emit neither neutrons nor gamma rays and can occur with either normal water or heavy water.

The boron impurity may be interstitial or it may collect in grain boundaries. The reaction may only occur if the boron is in one or the other of these distributions. It may also only occur when the amount of impurity falls within some narrow range. Thus, a palladium rod, may become “exhausted” after some time of operation if the boron concentration falls below some given limiting concentration.

Perhaps the worst indictment of the P&F experiment is its irreproducibility. No one has claimed to have seen the large heat production reported from Utah. Pons himself states that his experiment will only work occasionally—he claims that there is *live* palladium and *dead* palladium.

This could be interesting. Hydrogen absorbed in metals is known to accumulate in imperfections in the crystal lattice. It is possible that such defects promote the high concentrations of deuterium necessary to trigger the reaction.

I still have an old issue of the CRC handbook that lists the thermoelectric power of silicon as both +170 mV/K and -230 mV/K. How can it be both positive and negative? Notice that the determination of the sign of the Seebeck effect is trivial; this cannot be the result of an experimental error. In both cases “chemically pure” silicon was used. So, how come? We have a good and classical example of irreproducibility. That was back in the 1930s. Now any EE junior knows that one sample must have been *p*-silicon, while the other, *n*-silicon. Both could be “chemically pure”—to change the Seebeck sign, all it takes is an impurity concentration of 1 part in 10 million. Is there an equally subtle property in the palladium that will allow fusion in some cases?

In April, 1992, Akito Takahashi of Osaka University revealed that his cold fusion cell produced an average excess heat of 100 W over periods of months. The electric power fed to the cell was only 2.5 W. The main difference between the Takahashi cell and that of other experimenters is the use of palladium sheets (instead of rods) and of varying current to cause the cell to operate mostly under transient conditions. The excess heat measured is far too large to be attributed to errors in calorimetry. Disturbing to theoreticians is the absence of detectable neutrons. See D. H. Freedman’s (1992) report.

In spite being saddled with the stigma of “pseudo-science”, cold fusion does no seem to go away. The September 2004 issue of *IEEE Spectrum*, published a report titled “Cold Fusion Back from the Dead,” in which recent work on the cold fusion by reputable laboratories is mentioned. It quotes the US Navy as revealing that the Space and Naval Systems Center (San Diego) was working on this subject.<sup>†</sup> It also mentioned the Tenth International Conference on Cold Fusion that took place in Cambridge, MA in August 2003.

It appears that by 2004, “a number of groups around the world have reproduced the original Pons-Fleishmann excess heat effect . . .” *Mike McKubre* of SRI International maintains that the effect requires that the palladium electrode be 100% packed with deuterium (One deuterium-to-one-palladium atom). This coincides with our wild guess at the beginning of this sub-section.

At the moment, cold fusion research has gone partially underground, at least as far as the media are concerned. Yet, the consensus is that it

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<sup>†</sup> It is reported that Stanislaw Szpak, of the SNSC, has taken infrared pictures of miniexplosions on the surface of the palladium, when cold fusion appears to be taking place.

merits further study. This is also the opinion of independent scientists such as Paul Chu and Edward Teller who have been brought in as observers. It may be that cold fusion will one day prove practical. That is almost too good to be true and, for the classical fusion researchers, almost too bad to be true.

## 1.12 Financing

Some of the proposed alternative energy sources, such as the fusion reactor, require, for their implementation, a scientific break-through. Others need only technological development, as is the case of wind turbines or of ocean thermal energy converters. Still others have reached a fairly advanced stage of development, but their massive implementation awaits more favorable economic conditions, such as further increase in the price of oil. The production of synthetic fuel from coal falls in this category, as does the utilization of shale.

Finding new sources of energy is not difficult. What is difficult is finding new sources of *economically attractive* energy. It is, therefore, important to estimate the cost of the energy produced by different methods. One of the main ingredients of the cost formula is the cost of financing, examined below.

Frequently, the financing of the development is borne by the government, especially during the early high-risk stages of the work. It is an important political decision for the nation to finance or not to finance the development of a new energy source. For instance, the Solar Power Satellite scheme is one that has possibilities of being economical. However, its development costs, estimated as nearly 80 billion dollars, are too high to be funded by private corporations. Thus, the SPS system will be implemented only if the government feels justified in paying the bill.

Financing the implementation is simpler. Engineers can estimate roughly how the investment cost will affect the cost of the product by using a simple rule of thumb:

“The yearly cost of the investment can be taken as 20%<sup>†</sup> of the overall amount invested.”

Thus, if a 1 million dollar power plant is to be built, one must include in the cost of the generated energy, a sum of \$200,000 per year.

To allow a comparison of the costs of energy produced by different alternative sources, the Department of Energy has recommended a standard method of calculating the cost of the capital investment.

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<sup>†</sup> This percentage is, of course, a function of the current interest rate. In the low interest rate regimen of the early years of this millennium, the percentage is lower than 20%.

We will here derive an expression for the cost of a direct reduction loan.

Assume that the payment of the loan is to be made in  $N$  equal installments. We will consider a \$1.00 loan. Let  $x$  be the interest rate of one payment period (say, one month) and let  $p$  be the value of the monthly payment. At the end of the first month, the amount owed is

$$1 + x - p \quad (29)$$

and, at the end of the second month, it is

$$(1 + x - p)(1 + x) - p = (1 + x)^2 - p(1 + 1 + x) \quad (30)$$

and, at the end of the third month, it is

$$\begin{aligned} & [(1 + x)^2 - p(1 + 1 + x)](1 + x) - p \\ &= (1 + x)^3 - p[1 + (1 + x) + (1 + x)^2]. \end{aligned} \quad (31)$$

At the end of  $N$  months, the amount owed is zero because the loan has been repaid. Thus,

$$(1 + x)^N - p[1 + (1 + x) + (1 + x)^2 + \dots + (1 + x)^{N-1}] = 0, \quad (32)$$

whence

$$p = \frac{1}{(1 + x)^1 + (1 + x)^{-2} + \dots + (1 + x)^{-N}} = \left[ \sum_{\gamma=1}^N z^\gamma \right]^{-1}, \quad (33)$$

where

$$z \equiv (1 + x)^{-1}. \quad (34)$$

But

$$\sum_{\gamma=1}^N z^\gamma = \frac{1 - z^{N+1}}{1 - z} - 1, \quad (35)$$

hence

$$p = \frac{1 - z}{z - z^{N+1}} = \frac{x}{1 - (1 + x)^{-N}}. \quad (36)$$

The formula above yields the magnitude of the monthly payment as a function of the interest rate (per month) and the number of payments.

As an example, consider a small entrepreneur who owns a Diesel-electric generating plant in which he has invested \$1000 per kW. The utilization factor is 50%—that is, 4380 kWh of electricity are produced yearly for each kW of installed capacity. Taxes and insurance amount of

\$50 year<sup>-1</sup> kW<sup>-1</sup>. Fuel, maintenance and personnel costs are \$436 kW<sup>-1</sup> year<sup>-1</sup>. In order to build the plant, the entrepreneur borrowed money at 12% per year and is committed to monthly payments for 10 years. What is the cost of the generated electricity?

The monthly rate of interest is

$$(1 + x)^{12} = 1.12 \quad \therefore \quad x = 0.009489. \quad (37)$$

The number of payments is

$$N = 10 \text{ years} \times 12 \text{ months/year} = 120. \quad (38)$$

The monthly payment is

$$p = \frac{0.009489}{1 - (1 + 0.009489)^{-120}} = \$0.013995 \text{ month}^{-1}. \quad (39)$$

The yearly payment is

$$P = 12p = \$0.167937 \text{ year}^{-1}. \quad (40)$$

If there were no interest, the yearly payment would be \$0.1. Thus, the yearly cost of interest is \$0.067937.

All the above is on a loan of \$1.00. Since the plant cost \$1000 kW<sup>-1</sup>, the cost of the investment is \$167.94 kW<sup>-1</sup> year<sup>-1</sup>. But, on a per kW basis, there is an additional expense of \$50 for taxes and insurance, raising the yearly total to \$217.94. Thus, in this example, the yearly investment cost is 21.79% of the total amount.

A total of 4380 kWh per kW installed are generated (and sold) per year. The fixed cost per kWh is, therefore

$$\frac{217.94}{4380} = 0.0497 \text{ \$ kWh}^{-1}, \quad (41)$$

whereas, the fuel, maintenance and personnel cost is

$$\frac{436}{4380} = 0.0995 \text{ \$ kWh}^{-1}. \quad (42)$$

Total cost is 0.1492 \$ kWh<sup>-1</sup>. This is commonly expressed as 149.2 mils/kWh, an awkward unit. It is better to use 149.2 \$/MWh or, to stick to the conventional SI units of measure, \$41.4 GJ<sup>-1</sup>.

When the loan is paid after 10 years, does the entrepreneur own the plant? Maybe. The Diesel-generator may have only a 10-year life and a new one may have to be acquired.

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Abundant statistical information on energy: <http://www.eia.doe.gov/>

For more detailed information on some topics in this chapter, read:  
Sørensen, Bent, Renewable energy, *Academic Press* **2003**.



## PROBLEMS

1.1 Assume that from 1985 on the only significant sources of fuel are:

1. coal (direct combustion)
2. oil,
3. synthetic liquid fuel (from coal), and
4. natural gas.

Sources a, b and c are assume to follow the market penetration rule:

$$\ln \frac{f}{1-f} = at + b$$

where  $f$  is the fraction of the market supplied by the fuel in question and  $t$  is the year (expressed as 1988, for instance, not as simply 88). The coefficients are:

	$a$	$b$
for coal:	-0.0475,	92.14;
for oil:	-0.0436,	86.22.

The above coefficients are derived from historical data up to 1975.

The objective of this exercise is to predict what impact the (defunct) federal coal liquefaction program would have had on the fuel utilization pattern.

According to the **first in, first out** rule, the “free” variable, i.e., the one that does not follow the market penetration rule, is the natural gas consumption fraction,  $f_{ng}$ . The questions are:

- in what year will  $f_{ng}$  peak?
- what is the maximum value of  $f_{ng}$ ?

Assume that  $f_{syn}$  (the fraction of the market supplied by synthetic fuel) is 0.01 in 1990 and 0.0625 in 2000. Please comment.

1.2 The annual growth rate of energy utilization in the world was 3.5% per year in the period between 1950 and 1973. How long would it take to consume all available resources if the consumption growth rate of 3.5% per year is maintained?

Assume that the global energy resources at the moment are sufficient to sustain, at the current utilization rate

- a. 1000 years,
- b. 10000 years.

1.3 A car moves on a flat horizontal road with a steady velocity of 80 km/h. It consumes gasoline at a rate of 0.1 liters per km. Friction of the tires on the road and bearing losses are proportional to the velocity and, at

80 km/h, introduce a drag of 222 N. Aerodynamic drag is proportional to the square of the velocity with a coefficient of proportionality of 0.99 when the force is measured in N and the velocity in m/s.

What is the efficiency of fuel utilization? Assuming that the efficiency is constant, what is the “kilometrage” (i.e., the number of kilometers per liter of fuel) if the car is driven at 50 km/h?

The density of gasoline is 800 kg per cubic meter and its heat of combustion is 49 MJ per kg.

1.4 Venus is too hot, in part because it is at only 0.7 AU from the sun. Consider moving it to about 0.95 AU. One AU is the distance between Earth and Sun and is equal to 150 million km.

To accomplish this feat, you have access to a rocket system that converts mass into energy with 100% efficiency. Assume that all the energy of the rocket goes into pushing Venus. What fraction of the mass of the planet would be used up in the project? Remember that you are changing both kinetic and potential energy of the planet.

1.5 Consider the following arrangement:

A bay with a narrow inlet is dammed up so as to separate it from the sea, forming a lake. Solar energy evaporates the water causing the level inside the bay to be  $h$  meters lower than that of the sea.

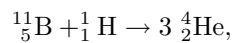
A pipeline admits sea water in just the right amount to compensate for the evaporation, thus keeping  $h$  constant (on the average). The inflow water drives a turbine coupled to an electric generator. Turbine plus generator have an efficiency of 95%.

Assume that there is heat loss neither by conduction nor by radiation. The albedo of the lake is 20% (20% of the incident radiation is reflected, the rest is absorbed). The heat of vaporization of water (at STP) is 40.6 MJ per kilomole. Average solar radiation is 250 W/square meter.

If the area of the lake is 100 km<sup>2</sup>, what is the mean electric power generated? What is the efficiency? Express these results in terms of  $h$ .

Is there a limit to the efficiency? Explain.

1.6 The thermonuclear (fusion) reaction



is attractive because it produces essentially no radiation and uses only common isotopes.

How much energy does 1 kg of boron produce? Use the data of Problem 1.11.

1.7 The efficiency of the photosynthesis process is said to be below 1% (assume 1%). Assume also that, in terms of energy, 10% of the biomass

1.42

produced is usable as food. Considering a population of 6 billion people, what percentage of the **land** area of this planet must be planted to feed these people.

1.8 Each fission of  $^{235}\text{U}$  yields, on average, 165 MeV and 2.5 neutrons. What is the mass of the fission products?

1.9 There are good reasons to believe that in early times, the Earth's atmosphere contained no free oxygen.

Assume that all the oxygen in the Earth's atmosphere is of photosynthetic origin and that all oxygen produced by photosynthesis is in the atmosphere. How much fossil carbon must there be in the ground (provided no methane has evaporated)? Compare with the amount contained in the estimated reserves of fossil fuels. Discuss the results.

1.10 What is the total mass of carbon in the atmosphere?

$\text{CO}_2$  concentration is currently 330 ppm but is growing rapidly!

If all the fossil fuel in the estimated reserves (see Section 1.8) is burned, what will be the concentration of  $\text{CO}_2$

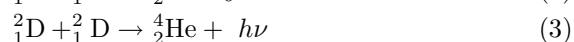
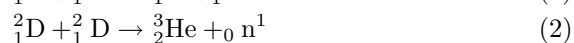
1.11 Here are some pertinent data:

Particle	Mass (daltons)	Particle	Mass (daltons)
electron	0.00054579903	alpha	4.001506175
muon	0.1134381	$^5_3\text{Li}$	5.01254
proton	1.007276467	$^6_3\text{Li}$	6.015122794
neutron	1.008664909	$^7_3\text{Li}$	7.01600455
$^1_1\text{H}$	1.007825032	$^{10}_5\text{B}$	10.012937
$^2_1\text{D}$	2.014101778	$^{11}_5\text{B}$	11.009305
$^3_1\text{T}$	3.016049278		
$^3_2\text{He}$	3.016029319		
$^4_2\text{He}$	4.002603254		
Constants			
$c$	$2.998 \times 10^8 \text{ m/s}$		
$h$	$6.625 \times 10^{-34} \text{ joule-sec}$		

To convert daltons to kg, divide by  $6.02213670 \times 10^{26}$ .

Deuterium is a very abundant fusion fuel. It exists in immense quantities in earth's oceans. It is also, relatively easy to ignite. It can undergo

three different reactions with itself:



For each reaction, calculate the energy released and, assuming equipartition of momenta of the reaction products, the energy of each product.

What is the energy of the photon released in Reaction 3?

1.12 Random access memories (RAMs) using the “Zing Effect” were first introduced in 1988 but only became popular in 1990 when they accounted for 6.3% of total RAM sales. In 1994 they represented \$712 million of a total of \$4.75 billion. Sales of all types of RAMs reached \$6 billion in 1997.

A company considering the expansion of Z-RAM production needs to have an estimate of the overall (all manufacturers) sales volume of this type of memory in the year 2000. Assume that the growth rate of the overall dollar volume of RAM sales between 1900 and 2000 is constant (same *percentage* increase every year)

1.13 A 1500-kg Porsche 912 was driven on a level highway on a windless day. After it attained a speed of 128.7 km/h it was put in neutral and allowed to coast until it slowed down to almost standstill. The coasting speed was recorded every 10 seconds and resulted in the table below.

From the given data, derive an expression relating the decelerating force to the velocity.

Calculate how much horse power the motor has to deliver to the wheel to keep the car at a constant 80 mph.

Coasting time (s)	Speed (km/h)	Coasting time (s)	Speed (km/h)
0	128.7	100	30.6
10	110.8	110	25.9
20	96.2	120	20.4
30	84.0	130	16.2
40	73.0	140	12.2
50	64.2	150	9.2
60	56.4	160	5.1
70	48.0	170	2.0
80	41.8	180	0
90	35.8		

1.14 The California Air Resources Board (CARB) mandated, for 1995, an upper limit of 200 g/km for the emission of CO<sub>2</sub> from a minivan.

This could be achieved by bubbling the exhaust through a Ca(OH)<sub>2</sub> bath or through a similar CO<sub>2</sub> sequestering substance. However, this solution does not seem economical. Assume that all the produced CO<sub>2</sub> is released into the atmosphere.

1.44

What is the minimum mileage (miles/gallon) that a minivan had to have by 1995. Assume gasoline is pentane ( $C_5H_{12}$ ) which has a density of  $626 \text{ kg m}^{-3}$ . A gallon is 3.75 liters and a mile is 1609 meters. The atomic mass of H is 1, of C is 12, and of O is 16.

1.15 A geological survey revealed that the rocks in a region of Northern California reach a temperature of 600 C at a certain depth. To exploit this geothermal source, a shaft was drilled to the necessary depth and a spherical cave with 10 m diameter was excavated. Water at 30 C is injected into the cave where it reaches the temperature of 200 C (still in liquid form, owing to the pressure) before being withdrawn to run a steam turbine.

Assume that the flow of water keeps the cave walls at an uniform 200 C. Assume, furthermore that, at 100 m from the cave wall, the rocks are at their 600 C temperature. Knowing that the heat conductivity,  $\lambda$ , of the rocks is  $2 \text{ W m}^{-1}\text{K}^{-1}$ , what is the flow rate of the water?

The heat capacity of water is  $4.2 \text{ MJ m}^{-3} \text{ K}^{-1}$  and the heat power flux ( $\text{W m}^{-2}$ ) is equal to the product of the heat conductivity times the temperature gradient.

1.16 The following data are generally known to most people:

- a. The solar constant,  $C$  (the solar power density), at earth's orbit is  $1360 \text{ W m}^{-2}$ ;
- c. the astronomical unit (AU, the average sun-earth distance) is about 150 million km;
- c. the angular diameter of the moon is  $0.5^\circ$ .

Assume that the sun radiates as a black body. From these data, estimate the sun's temperature.

1.17 Using results from Problem 1.16, compare the sun's volumetric power density (the number of watts generated per  $\text{m}^3$ ) with that of a typical homo sapiens.

1.18 Pollutant emission is becoming progressively the limiting consideration in the use of automobiles. When assessing the amount of pollution, it is important take into account not only the emissions from the vehicle but also those resulting from the fuel production processes. Gasoline is a particularly worrisome example. Hydrocarbon emission at the refinery is some 4.5 times larger than that from the car itself. Fuel cell cars (see Chapter 9) when fueled by pure hydrogen are strictly a zero emission vehicle. However, one must inquire how much pollution results from the production of the hydrogen. This depends on what production method is used (see Chapter 10). The cheapest hydrogen comes from reforming fossil fuels and that generates

a fair amount of pollution. A clean way of producing hydrogen is through the electrolysis of water; but, then, one must check how much pollution was created by the generation of the electricity. Again, this depends on how the electricity was obtained: if from a fossil fuel steam plant, the pollution is substantial, if from hydroelectric plants, the pollution is zero.

The technical means to build and operate a true zero emission vehicle are on hand. This could be done immediately but would, at the present stage of the technology, result in unacceptably high costs.

Let us forget the economics and sketch out roughly one possible ZEV combination. Consider a fuel-cell car using pure hydrogen (stored, for instance, in the form of a hydride—Chapter 11). The hydrogen is produced by the electrolysis of water and the energy required for this is obtained from solar cells (Chapter 14). Absolutely no pollution is produced. The system is to be dimensioned so that each individual household is independent. In other words, the solar cells are to be installed on the roof of each home.

Assume that the car is to be driven an average of 1000 miles per month and that its gasoline driven equivalent can drive 30 miles/gallon. The fuel cell version, being much more efficient, will drive 3 times farther using the same energy as the gasoline car.

How many kilograms of hydrogen have to be produced per day?

How large an area must the solar cell collector have?

You must make reasonable assumptions about the solar cell efficiency, the efficiency of the electrolyzer and the amount of insolation (Chapter 12).

#### 1.19 From a fictitious newspaper story:

*A solar power plant in the Mojave Desert uses 1000 photovoltaic panels, each “40 meters square.” During the summer, when days are invariably clear, the monthly sale of electricity amounts to \$22,000. The average price charged is 3 cents per kWh. The plant is able to sell all the electricity produced.*

There is an unfortunate ambiguity in the story: “40 meters square” can be interpreted as a square with 40 meters to its side or as an area of 40 m<sup>2</sup>.

From the data in the story, you must decide which is the correct area.

#### 1.20 Sport physiologists have a simple rule of thumb: Any healthy person uses about 1 kilocalorie per kilometer per kilogram of body weight when running.

It is interesting to note that this is true independently of how well trained the runner is. A trained athlete will cover 1 km in much less time than an occasional runner but will use about the same amount of energy. Of course, the trained athlete uses much more power.

The overall efficiency of the human body in transforming food intake into mechanical energy is a (surprisingly high) 25%!

A good athlete can run 1 (statute) mile in something like 4 minutes and run the Marathon (42.8 km) in a little over 2 hours.

1. Calculate the power developed in these races. Repeat for a poor performer who runs a mile in 8 minutes and the Marathon in 5 hours. Assume a body weight of 70 kg.
2. Evaporation of sweat is the dominant heat removal mechanism in a human body. Is this also true for a dog? For a horse?
3. Assuming that all the sweat evaporates, i.e., none of it drips off the body, how much water is lost by the runners in the four cases above? The latent heat of vaporization of water is 44.1 MJ/kmole.

1.21 One major ecological concern is the emission of hot-house gases, the main one being CO<sub>2</sub>.

A number of measures can be taken to alleviate the situation. For instance, the use of biomass derived fuels does not increase the carbon dioxide content of the atmosphere.

Fossil fuels, on the other hand are a major culprit. Suppose you have the option of using natural gas or coal to fire a steam turbine to generate electricity. Natural gas is, essentially, methane, CH<sub>4</sub>, while coal can be taken (for the purposes of this problem only) as eicosane, C<sub>20</sub>H<sub>42</sub>. The higher heat of combustion of methane is 55.6 MJ/kg and that of eicosane is 47.2 MJ/kg.

For equal amounts of generated heat, which of the two fuels is preferable from the CO<sub>2</sub> emission point of view? What is the ratio of the two emission rates?

1.22 A planet has a density of 2500 kg/m<sup>3</sup> and a radius of 4000 km. Its “air” consists of 30% ammonia, 50% carbon dioxide and 20% nitrogen.

Note that the density,  $\delta_{earth}$ , of Earth is 5519 kg/m<sup>3</sup>.

What is the acceleration of gravity on the surface of the planet?

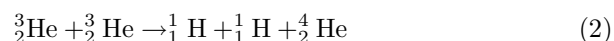
1.23 At 100 million km from a star, the light power density is 2 kW/m<sup>2</sup>. How much is the total insolation on the planet of Problem 1.22 if it is 200 million km from the star. The total insolation on earth is 173,000 TW.

1.24 <sup>3</sup>He can be used as fuel in “dream” fusion reactions—that is, in reactions that involve neither radioactive materials nor neutrons. Two possible reactions are



1.47

and



- 1 For each of the above reactions, calculate the energy (in kWh) released by 1 kg of  ${}^3_2\text{He}$ .

On earth,  ${}^3_2\text{He}$  represents 0.00013% of the naturally occurring helium. The US helium production amounts, at present, to 12,000 tons per year.

2. If all this helium were processed to separate the helium-three, what would be the yearly production of this fuel?

There are reasons to believe that there is a substantial amount of  ${}^3_2\text{He}$  on the moon. Let us do a preliminary analysis of the economics of setting up a mining operation on our satellite.

One of the advantages of using “dream” reactions is that only charged particles (protons and alphas) are produced. The energy associated with charged particles can be more efficiently be transformed into electricity than when the energy is carried by neutrons which must first produce heat that is then upgraded to mechanical and electric energy by inefficient heat engines. Thus, it is not necessarily optimistic to assign a 30% efficiency for the conversion of fusion energy into electricity.

3. How many kWh of electricity does 1 kg of  ${}^3_2\text{He}$  produce? Use the most economical of the two reactions mentioned.

Assume that the plant factor is 70% (the reactor delivers, on average, 70% of the energy it would deliver if running constantly at full power). Assume further that the cost of the fusion reactor is \$2000/kW and that the cost of borrowing money is 10% per year. Finally, the cost of running the whole operation is \$30 kW<sup>-1</sup>year<sup>-1</sup>.

4. How much would the electricity cost (per kWh) if the fuel were free?
5. How much can we afford to pay for 1 kg of  ${}^3_2\text{He}$  and still break even when electricity is sold at 5 cents per kWh?

1.25 Between 1955 and 1995, the ocean temperature (Atlantic, Pacific, and Indian) increased by 0.06 C.

Estimate how much energy was added to the water.

What percentage of the solar energy incident on earth during these 40 years, was actually retained by the ocean?

1.26 There seems to be a possibility that climate changes will cause the polar ice caps to melt. The amount of ice in Antarctica is so large that if it

1.48



were to melt, it would submerge all port cities such as New York and Los Angeles.

Estimate by how much the sea level would rise if only the north pole ice is melted leaving Greenland and Antarctica untouched.

1.27 Refueling a modern ICV with 50 liters of gasoline may take, say, 5 minutes. A certain amount of energy was transferred from the pump to the car in a given time. What is the power represented by this transfer? Assume that the overall efficiency of a gasoline car is 15% and that of an electric car is 60%. How much power is necessary to charge the batteries of the electric car in 5 minutes (as in the ICV case)? Assume that the final drive train energy is the same in both the ICV and the EV. Is it practical to *recharge* a car as fast as *refueling* one?

1.28 Some of the more attractive fuels happen to be gases. This is particularly true of hydrogen. Thus, storage of gases (Chapter 11) becomes an important topic in energy engineering. Lawrence Livermore Labs, for instance, has proposed glass micro-balloons, originally developed for housing minute amounts of tritium-deuterium alloy for laser fusion experiments. When heated, the glass becomes porous and hydrogen under pressure can fill the balloons. Cooled, the gas is trapped.

Clathrate is one of nature's way of storing methane, even though no one is proposing it as a practical method for transporting the gas.

Methane clathrate frequently consists of cages of 46  $\text{H}_2\text{O}$  trapping 8  $\text{CH}_4$  molecules.

1. What is the gravimetric metric concentration,  $GC$ , of methane in the clathrate? Gravimetric concentration is the ratio of the mass of the stored gas to the total mass of gas plus container.

Consider a hermetic container with  $1 \text{ m}^3$  internal volume, filled completely with the clathrate described which has a density of  $900 \text{ kg/m}^3$ . Assume that by raising the temperature to 298 K, the material will melt and methane will evolve. Assume also (although this is not true) that methane is insoluble in water.

2. What is the pressure of the methane in the container?

1.29 A Radioisotope Thermal Generator (RTG) is to deliver 500 W of dc power to a load at 30 V. The generator efficiency (the ratio of the dc power out to the heat power in) is 12.6%. The thermoelectric generator takes heat in at 1200 K and rejects it at 450 K. The heat source is plutonium-241. This radioactive isotope has a half-life of 13.2 years and decays emitting  $\alpha$  and  $\beta^-$  particles. These particles have an aggregate energy of 5.165 MeV.

Only 85% of the power generated by the plutonium finds its way to the thermoelectric generator. The rest is lost.

How many kilograms of plutonium are required? Note that radioactive substances decay at a rate proportional to the amount of undecayed substance and to a constant decay rate,  $\lambda$ :

$$\frac{dN}{dt} = -\lambda N.$$

1.30 In the USA we burn (very roughly) an average of 150 GW of coal, 40 GW of oil and 70 GW of natural gas.

Assume that

Coal is (say)  $C_{20}H_{44}$  and that it yield 40 MJ per kg,

Oil is (say)  $C_{10}H_{22}$  and yields 45 MJ per kg.

Natural gas is  $CH_4$  and yields 55 MJ per kg.

How many kg of carbon are released daily by the combustion of coal alone? (Clearly, after you have handled coal, the other two fuels can be handled the same way. But, for the sake of time, don't do it.)

1.50